引張り･圧縮

フックの法則と弾性係数

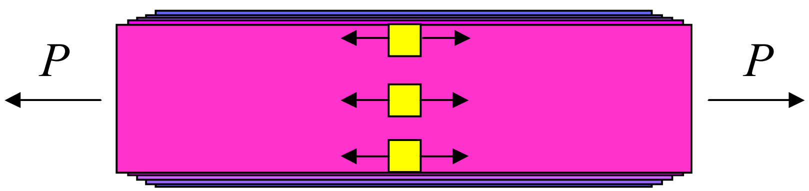
$$\sigma = E\varepsilon$$

σ :垂直応力

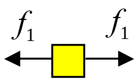
E :弾性係数

ε :垂直ひずみ

引張り･圧縮



単位断面における垂直応力



$$\sigma = \frac{f_1}{A_1}$$

$$\sigma \cdot A_1 = f_1$$

断面全体では

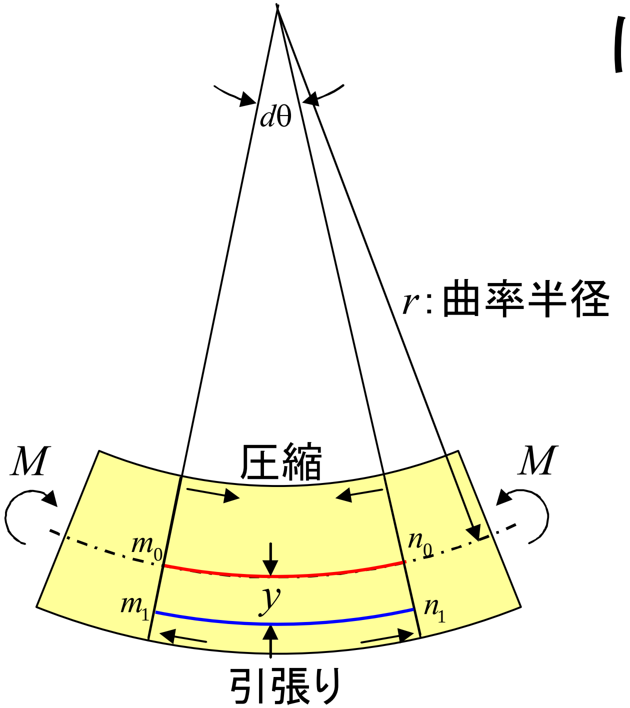
$$\int_A \sigma \cdot A_1 = \int_A f_1$$

$$\sigma \int_A A_1 = \int_A f_1$$

$$\sigma \cdot A = P$$

$$\sigma = \frac{P}{A}$$

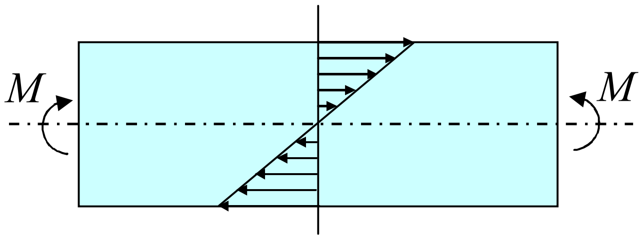
はりの曲げ



$m_1 n_1$ 面に生ずるひずみ ε_x は

$$\varepsilon_x = \frac{(m_1 n_1 - m_0 n_0)}{m_0 n_0} = \frac{(r+y)d\theta - rd\theta}{rd\theta} = \frac{y}{r}$$

曲げ応力分布

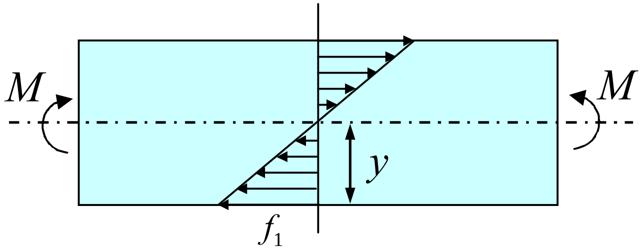


曲げ応力 σ_x は

$$\sigma_x = E \cdot \varepsilon_x = \frac{E}{r} y$$

はりの曲げ

曲げ応力分布



単位面積 A_1 あたりのモーメント

$$M_1 = f_1 \cdot y$$
$$= \sigma_x A_1 \cdot y$$
$$= \frac{E}{r} y^2 A_1$$

断面全体のモーメントは曲げモーメントに等しいので

$$\int_A M_1 = \int_A \frac{E}{r} y^2 A_1$$

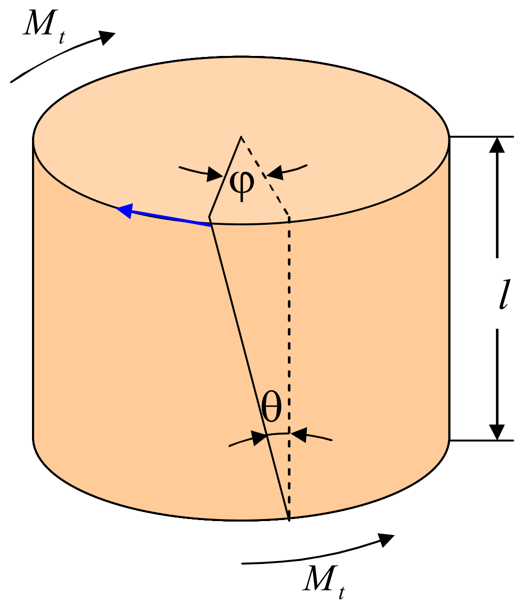
$$M = \frac{E}{r} \int_A y^2 A_1$$

I_z : 断面二次モーメント

$$\frac{E}{r} = \frac{M}{I_z}$$

$$\sigma_x = \frac{E}{r} y = \frac{My}{I_z}$$

ねじり（1）



せん断ひずみ

$$\gamma = \frac{\overset{\frown}{AA'}}{AB}$$

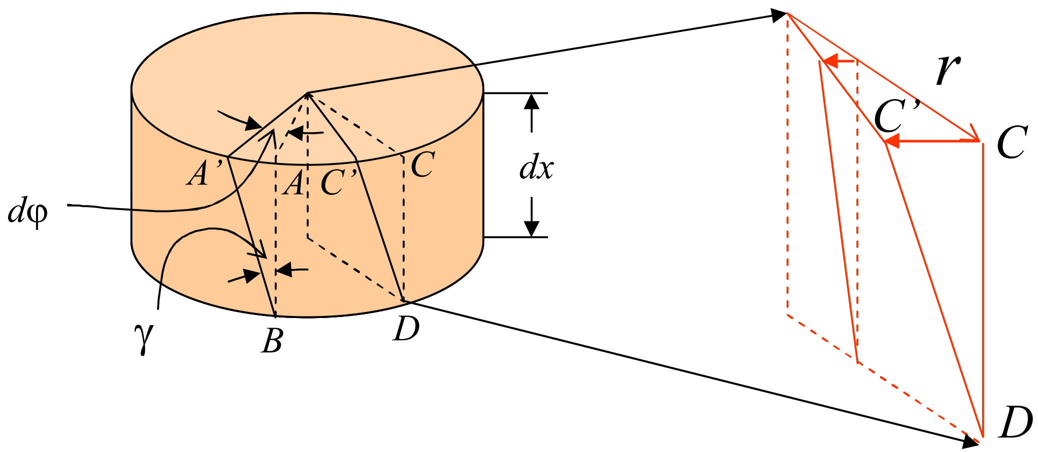
$\overset{\frown}{AA'} = rd\varphi \quad AB = dx$ より

$$\gamma = r\frac{d\varphi}{dx}$$

ここで $\frac{d\varphi}{dx} = \frac{\varphi}{l} = \theta$ ；比ねじり角

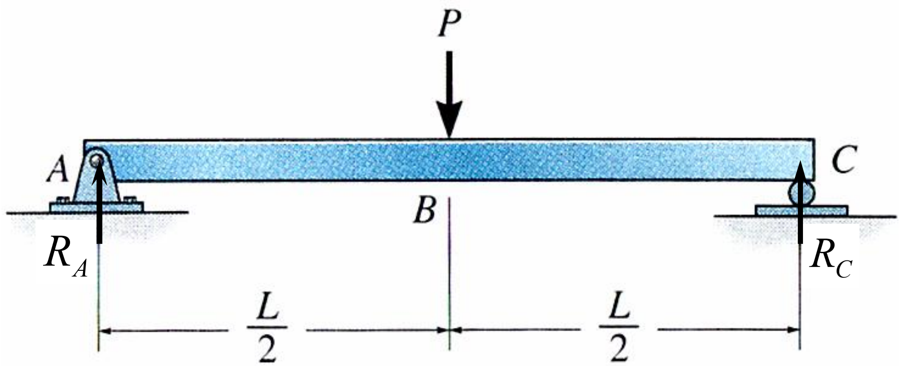
$$\therefore \gamma = r\theta$$

丸棒の一部である微小厚さdxの円板



はりの曲げ

Example 1
右図に示す両端支持はりのせん断力分布と曲げモーメント分布図を示せ。

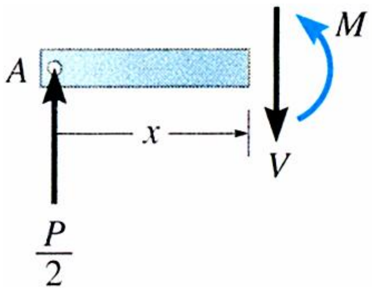


力の釣り合いより点A、点Cにおける反力R_A、R_Cは

力の釣り合いより　$$R_A = R_C = \frac{P}{2}$$

はりの左側部ABにおいて

支点Aから任意の距離xにおけるせん断力をV、曲げモーメントをMとすると



鉛直方向の力の釣り合いより

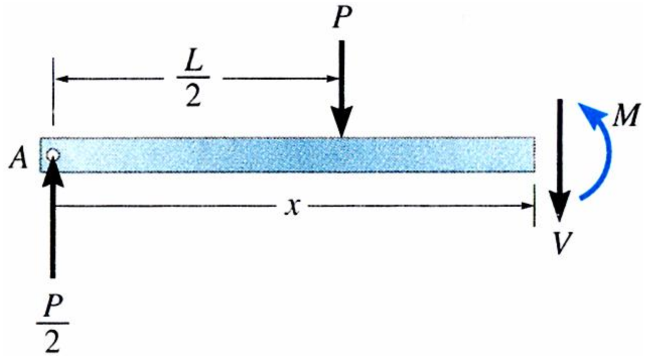
$$V = \frac{P}{2}$$

はりの先端周りのモーメントの釣り合いより

$$M = \frac{P}{2}x$$

はりの曲げ

はりの右側部ABにおいて



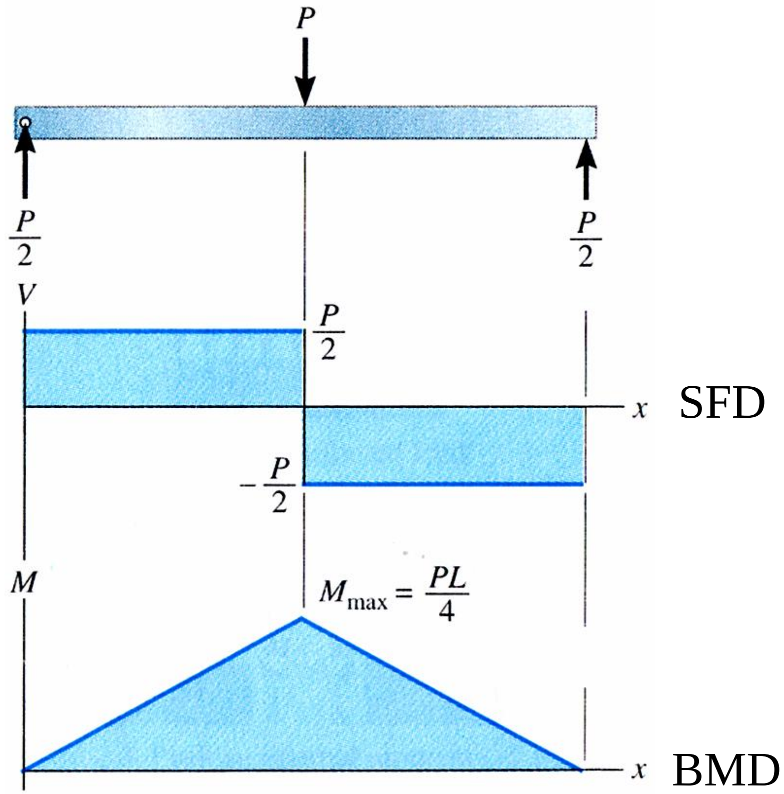
鉛直方向の力の釣り合いより

$$\frac{P}{2} - P - V = 0 \qquad \therefore V = -\frac{P}{2}$$

はりの先端におけるモーメントの釣り合いより

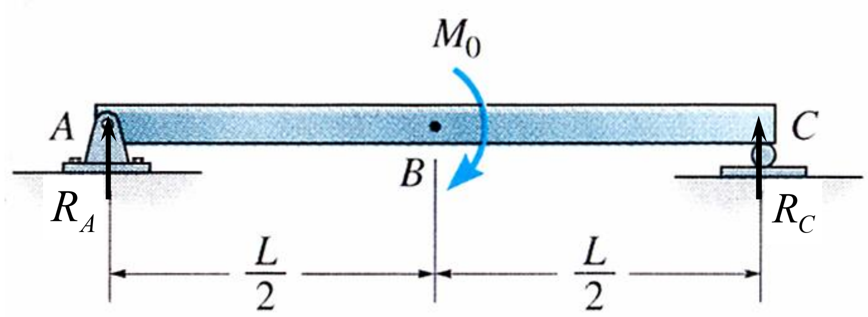
$$M + P\left(x - \frac{L}{2}\right) - \frac{P}{2}x = 0$$

$$\therefore M = \frac{P}{2}(L - x)$$



はりの曲げ

Example 2
右図に示す両端支持はりのせん断力分布と曲げモーメント分布図を示せ。



鉛直方向の力の釣り合いより

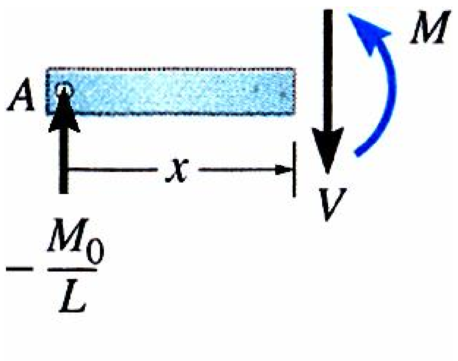
$$R_A + R_C = 0$$

B点まわりのモーメントの釣り合いより

$$-R_A \frac{L}{2} + R_C \frac{L}{2} - M_0 = 0$$

$$\therefore R_A = -\frac{M_0}{L} \quad R_C = \frac{M_0}{L}$$

はりの左側部ABにおいて



鉛直方向の力の釣り合いより

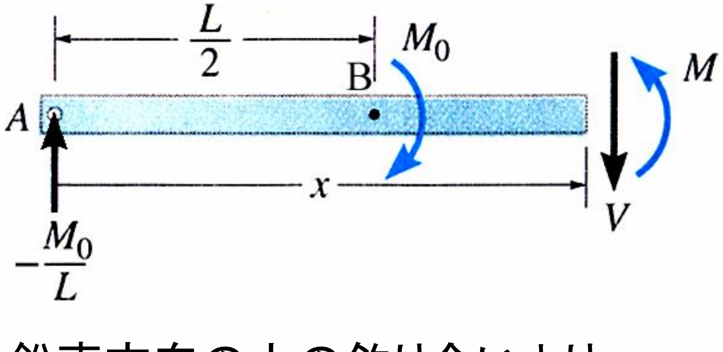
$$R_A - V = 0 \qquad \therefore V = -\frac{M_0}{L}$$

はり先端周りのモーメントの釣り合いより

$$-R_A x + M = 0 \qquad \therefore M = -\frac{M_0}{L} x$$

はりの曲げ

はりの右側部ABにおいて



鉛直方向の力の釣り合いより

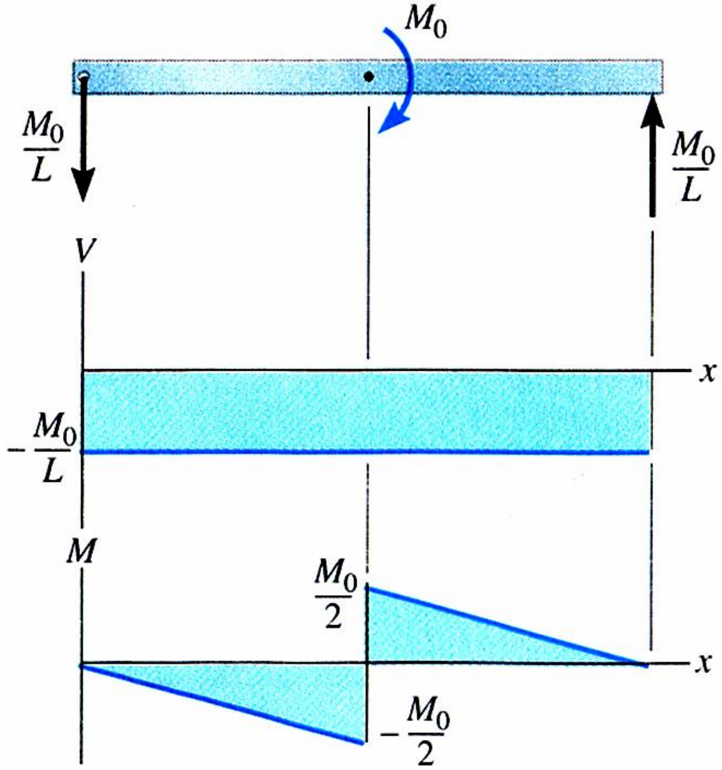
$$R_A - V = 0 \qquad \therefore V = -\frac{M_0}{L}$$

B点周りのモーメントの釣り合いより

$$-R_A \frac{L}{2} - V\left(x - \frac{L}{2}\right) - M_0 + M = 0$$

$$\therefore \quad M = M_0 - \frac{M_0}{L}x$$

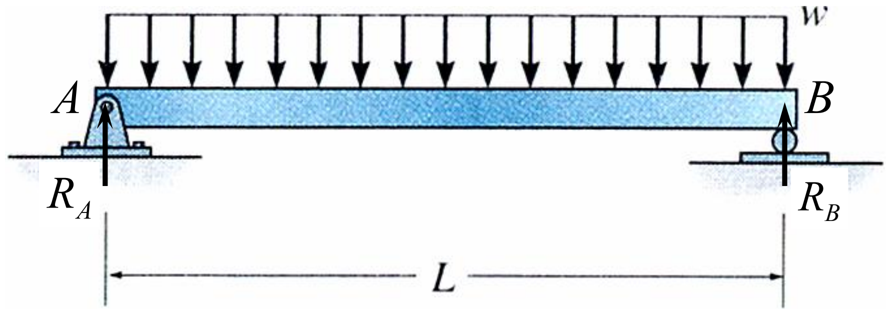
$$= M_0\left(1 - \frac{x}{L}\right)$$



はりの曲げ

Example 3

右図に示す両端支持はりのせん断力分布と曲げモーメント分布図を示せ。



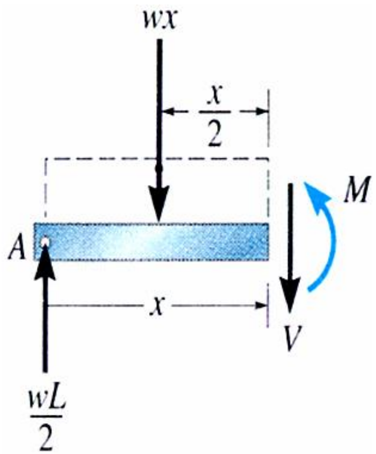
鉛直方向の力の釣り合いより

$$R_A + R_B - wL = 0$$

力の対称性より

$$R_A = R_B = \frac{wL}{2}$$

支点Aから任意の距離xで区切られたはりを考えると



力の釣り合いより

$$\frac{wL}{2} - wx - V = 0 \qquad \therefore V = w\left(\frac{L}{2} - x\right)$$

はり先端周りのモーメントの釣り合いより

$$-\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0 \qquad \therefore M = \frac{w}{2}\left(Lx - x^2\right)$$

はりの曲げ

せん断力と曲げモーメントは

$$\begin{cases} V = w\left(\dfrac{L}{2} - x\right) \\ M = \dfrac{w}{2}\left(Lx - x^2\right) \end{cases}$$

せん断力が0になるとき

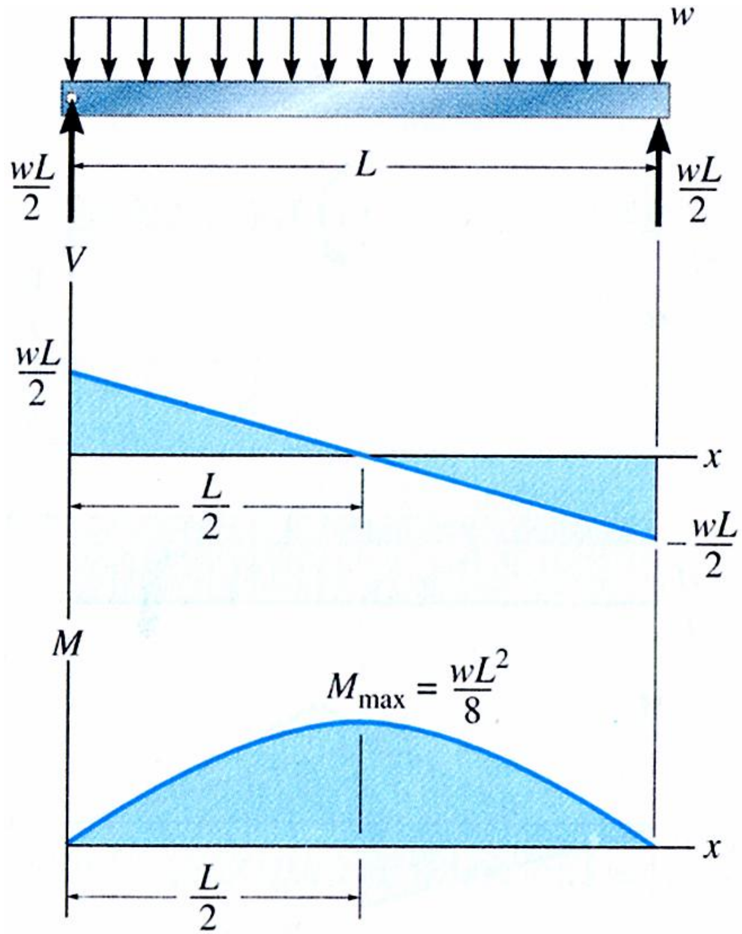
$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$\therefore x = \frac{L}{2}$$

曲げモーメントが最大になるのは

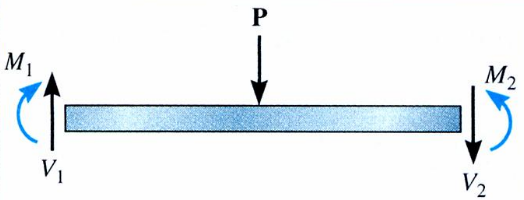
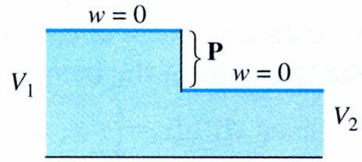
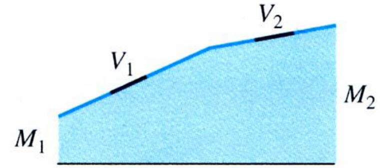
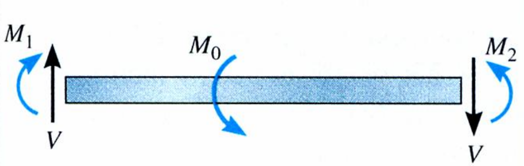
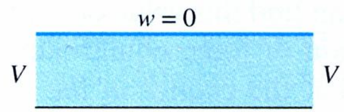
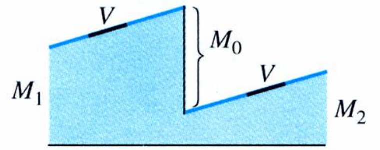
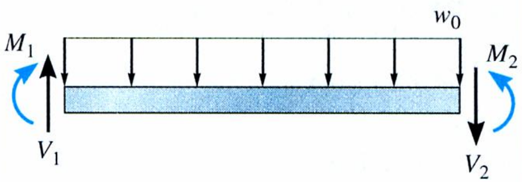
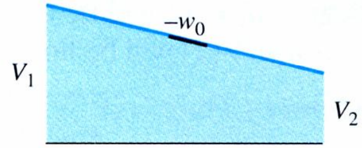
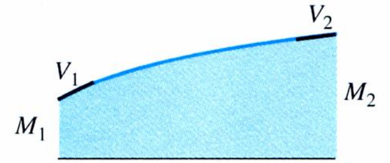
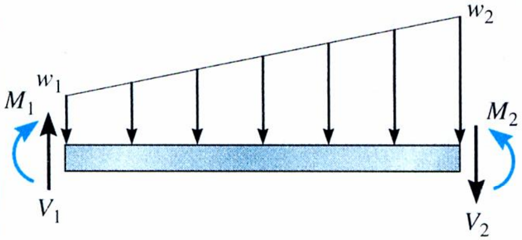
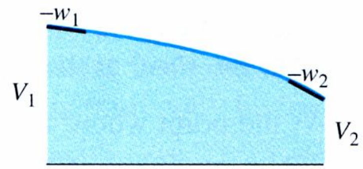
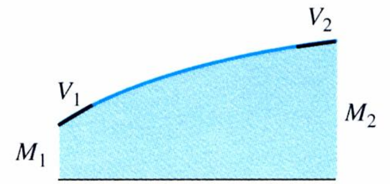
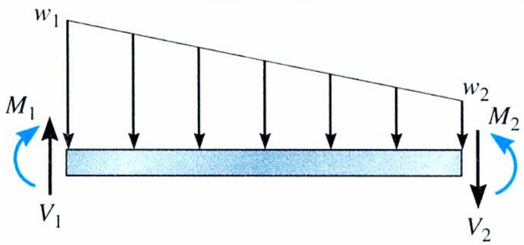
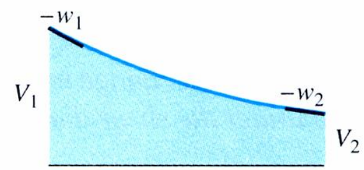
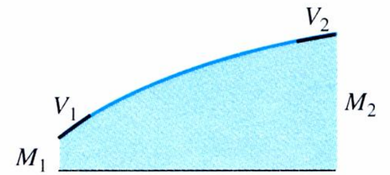
$$M_{\max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$



SFD

BMD

Loading	Shear Diagram $\frac{dV}{dx} = -w$	Moment Diagram $\frac{dM}{dx} = V$
	 Downward force **P** causes V to jump downward from V_1 to V_2.	 Constant slope changes from V_1 to V_2.
	 No change in shear since slope $w = 0$.	 Constant positive slope. Counterclockwise $\mathbf{M}_0$ causes M to jump downward.
	 Constant negative slope.	 Positive slope that decreases from V_1 to V_2.
	 Negative slope that increases from $-w_1$ to $-w_2$.	 Positive slope that decreases from V_1 to V_2.
	 Negative slope that decreases from $-w_1$ to $-w_2$.	 Positive slope that decreases from V_1 to V_2.