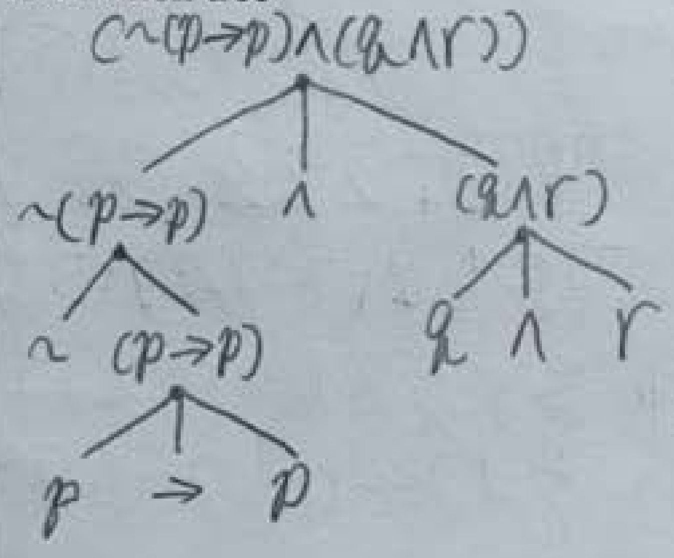
- Answer the questions below in Japanese or English. The language of statement logic that we worked with in class is called *language L* throughout the exam questions.

A. Draw a formation tree for (1) and construct its truth table. Provide the leftmost columns for the truth values of atomic statements and the final column. Additionally, say which type of statement (1) belongs to, a tautology, a contradiction, or a contingency.

(1) $(\sim(p \rightarrow p) \wedge (q \wedge r))$

Formation tree



Truth table

p	q	r	p→p	~(p→p)	(q∧r)	(~(p→p)∧(q∧r))
0	0	0	1	0	0	0
0	0	1	1	0	0	0
0	1	0	1	0	0	0
0	1	1	1	0	1	0
1	0	0	1	0	0	0
1	0	1	1	0	0	0
1	1	0	1	0	0	0
1	1	1	1	0	1	0

3

Statement type: a contradiction

B. Suppose that someone proposes (2) as the syntax of a language of statement logic. The language has the same vocabulary as language L, but its syntax is slightly different, as can be recognized in (2). **Say whether or not each of the strings of symbols given in (3) is a well-formed formula (wff) when (2) is assumed.** Also, **briefly discuss, in each case, whether the result obtained is good news or bad news for those who advocate (2)**, given that statement logic is a model of natural language.

(2) i. Any atomic statements are wff's.
ii. If X is an atomic statement, then $\sim X$ is a wff.
iii. If X and Y are atomic statements, then $(X \vee Y)$, $(X \wedge Y)$, $(X \rightarrow Y)$, and $(X \leftrightarrow Y)$ are all wff's.
iv. Nothing else is a wff.

(3) a. $\vee s$
b. $(\sim s \wedge t)$

	Well-formed?	Good result or bad result?
(3)a	wffではない	bad news
(3)b	wffではない	good news

2

C. (5) is the truth table for the Japanese sentence in (4). Based on what you think is the most natural interpretation of this Japanese sentence, complete the table by giving "1" or "0" in the rightmost column.

(4) 花子は先生がパーティに来ない時しかパーティに来ないなあ。　$p \vee q$　$\sim p$

(5)

	先生がパーティに	花子がパーティに	(4)
状況 1	来る	来る	0
状況 2	来る	来ない	1
状況 3	来ない	来る	1
状況 4	来ない	来ない	1

2

D. Translate (4) into statement logic, assuming that p and q are associated with Japanese sentences in the way shown in (6). Your answer must be consistent with your answer to Question C.

(6) p: 先生がパーティに来る。　q: 花子がパーティに来る。

$$q \rightarrow \sim p$$

/

E. The derivation in (7) is problematic. Discuss its problems.

(7)

1.	$(p \wedge \sim p) \rightarrow q$	Premise
2.	$\sim\sim r \rightarrow \sim(p \rightarrow q)$	Premise
3.	$r \rightarrow \sim(p \rightarrow q)$	2 Compl. [Sub.]
4.	$p \rightarrow q$	1 Simpl. [Sub.]
5.	$\sim r$	3, 4 M.T.

4. で1. にRules of Inference のSimplificationを(p∧~p)の部分のみに適用しているが、Rules of Inferenceは真である~~条件~~にしか適用できず、(p∧~p)が真であることは示されて行/全体いない。むしろ、Complement Lawsにより(p∧~p)は偽である。したがって、(7)の証明は不適切である。

1.

F. Passage (8) is from Partee et al. 1993, p. 121, where "the proof above" refers to the proof cited in (9). Explain the paragraph. Make your answer specific, as opposed to too general.

(8) Indirect proof can be shown to be a special form of conditional proof in the following way. Adding the auxiliary premise $\sim p$ in the proof above, for example, we derive $(r \;\&\; \sim r)$. By the rule of conditional proof we get $(\sim p \rightarrow (r \;\&\; \sim r))$. As the next line we add the tautology $\sim(r \;\&\; \sim r)$: a tautology can be written down as a valid step anywhere in any proof since it can never be false. Then we derive $\sim\sim p$ by *Modus Tollens* and then p by the Complement Law.

(9) Prove p from $(p \vee q)$, $(q \rightarrow r)$ and $\sim r$

1.	$p \vee q$	
2.	$q \rightarrow r$	
3.	$\sim r$	
4.	$\sim p$	Auxiliary Premise
5.	q	1, 4 D.S.
6.	r	2, 5 M.P.
7.	$r \;\&\; \sim r$	3, 6 Conj.
8.	p	4–7 Indirect Proof

間接的証明は、次のようにして条件付き証明の特別な形式であると見なすことができる。例えば、(9)のように~pを仮定して(r&~r)を示したとする。条件付き証明のルールによれば、これから(~p→(r&~r))が得られる。次に恒真文である~(r&~r)を与える。恒真文は偽になることが決してないため、いかなる証明にも有効な手順として書くことができる。そして、Modus Tollensによって~~pが示され、さらにComplement Lawによってpが示される。

3

G. Prove (10)a, b. Use conditional proof for (10)a. Give each line a justification.

(10)a. $p \rightarrow (q \wedge r)$

$\therefore (q \rightarrow s) \rightarrow (p \rightarrow s)$

1. $p \rightarrow (q \wedge r)$
2. $q \rightarrow s$

b. p

$p \rightarrow q$

$\sim r \rightarrow \sim q$

$\therefore \sim(p \rightarrow \sim r)$

1. p
2. $p \rightarrow q$
3. $\sim r \rightarrow \sim q$
4. q
5. $\sim\sim q$
6. $\sim\sim r$ 3, 5
7. $p \;\&\; \sim\sim r$
8. $\sim\sim p \;\&\; \sim\sim r$
9. $\sim(\sim p \vee \sim r)$
10. $\sim(p \rightarrow \sim r)$

(10)b.

1.	p	Premise
2.	$p \to q$	Premise
3.	$\sim r \to \sim q$	Premise
4.	q	2,1 M.P.
5.	$\sim\sim q$	4 Compl.
6.	$\sim\sim r$	3,5 M.T.
7.	$p \wedge \sim\sim r$	1,6 Conj.
8.	$\sim\sim p \wedge \sim\sim r$	7 Compl.[Sub]
9.	$\sim(\sim p \vee \sim r)$	8 DeM.
10	$\sim(p \to \sim r)$	9 Cond.[Sub]

2.

(10)a.

1.	$p \to (q \wedge r)$	Premise
2.	$q \to s$	Aux.Prem.
3.	$\sim p \vee (q \wedge r)$	1 Cond.
4.	$(\sim p \vee q) \wedge (\sim p \vee r)$	3 Distr.
5.	$\sim p \vee q$	4 Simp.
6.	$p \to q$	5 Cond.
7.	$p \to s$	7,2 H.S.
8.	$(q \to s) \to (p \to s)$	2-7 Conditional Proof

2.