$\boxed{6}$

(1) $\sin 3x = -4\sin^3 x + 3\sin x$ より

$$\int \cos x \sin 3x \, dx$$
$$= \int (-4\sin^3 x \cos x + 3\sin x \cos x) \, dx$$
$$= -\sin^4 x + \frac{3}{2}\sin^2 x + C$$ (C = 積分定数)

(2)
$$\cos^4 x = (1-\sin^2 x)\cos^2 x$$
$$= \cos^2 x - (\sin x \cos x)^2$$
$$= \frac{1+\cos 2x}{2} - \frac{1}{4}\sin^2 2x$$
$$= \frac{1+\cos 2x}{2} - \frac{1-\cos 4x}{8} = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

よって
$$\int \cos^4 x \, dx = \int \left(\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\right) dx$$
$$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$ (C = 積分定数)

(3)
$$\frac{1}{\sqrt{5-x^2}} + \frac{1}{\sqrt{x^2-3}}$$
$$= \frac{1}{\sqrt{5}\sqrt{1-\frac{x^2}{5}}} + \frac{1}{\sqrt{x^2-3}}$$

よって
$$\int \left(\frac{1}{\sqrt{5-x^2}} + \frac{1}{\sqrt{x^2-3}}\right) dx$$
$$= \frac{1}{\sqrt{5}}\int \frac{1}{\sqrt{1-\frac{x^2}{5}}} dx + \int \frac{1}{\sqrt{x^2-3}} dx$$

$t = \frac{x}{\sqrt{5}}$ とおくと

$dt = \frac{1}{\sqrt{5}} dx$

よって
$$\int \frac{1}{\sqrt{1-t^2}} dt + \int \frac{1}{\sqrt{x^2-3}} dx$$
$$= \sin^{-1} t + \log|x+\sqrt{x^2-3}| + C$$
$$= \sin^{-1}\frac{x}{\sqrt{5}} + \log|x+\sqrt{x^2-3}| + C$$

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(4) $\cos^5 x = \cos x\,(1-\sin^2 x)^2$

$t = \sin x$ とおくと $dt = \cos x\,dx$

$$\int \cos^5 x\,dx = \int \cos x\,(1-\sin^2 x)^2\,dx$$
$$= \int (1-t^2)^2\,dt$$
$$= \int (t^4 - 2t^2 + 1)\,dt$$
$$= \frac{t^5}{5} - \frac{2}{3}t^3 + t + C$$
$$= \frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x + C \quad (C = 積分定数)$$

(5) $t = \cos x$ とすると

$$\frac{dt}{dx} = -\sin x$$
$$\int \frac{\sin^3 x}{\cos x}\,dx = -\int \frac{\sin^3 x}{t}\cdot\frac{1}{\sin x}\,dt$$
$$= -\int \frac{\sin^2 x}{t}\,dt$$
$$= -\int \frac{1-t^2}{t}\,dt$$
$$= -\log|t| + \frac{t^2}{2} + C$$
$$= -\log|\cos x| + \frac{\cos^2 x}{2} + C \quad (C = 積分定数)$$

(別) (与式) $= \int \frac{(1-\cos^2 x)\sin x}{\cos x}\,dx$

$$= \int \frac{\sin x}{\cos x}\,dx - \int \sin x\cos x\,dx$$
$$= -\log|\cos x| - \frac{1}{2}\sin^2 x + C$$

(6) $t = \tan x$ とすると

$$dt = \frac{1}{\cos^2 x}\,dx$$
$$dx = \cos^2 x\,dt$$
$$= \frac{1}{1+t^2}\,dt$$

よって $\int \tan^2 x\,dx = \int \frac{t^3}{1+t^2}\,dt$

$$= \int t\,dt - \int \frac{t}{1+t^2}\,dt$$
$$= \frac{1}{2}t^2 - \frac{1}{2}\log(1+t^2) + C$$
$$= \frac{1}{2}\tan^2 x - \frac{1}{2}\log(1+\tan^2 x) + C$$
$$= \frac{1}{2}\tan^2 x + \log|\cos x| + C$$

(別) $\int \left(\frac{1}{\cos^2 x} - 1\right)\tan x\,dx$

$$= \int \frac{\tan x}{\cos^2 x} - \int \tan x\,dx \qquad \left(\frac{1}{2}\tan^2 x\right)'$$
$$= \int (\tan x)'\tan x - \int \frac{\sin x}{\cos x}\,dx$$
$$= \frac{1}{2}\tan^2 x + \log|\cos x| + C$$

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(7) $\int \sqrt{\sin x}\,\cos^3 x\,dx$

$= \int \sqrt{\sin x}\,(1-\sin^2 x)\cos x\,dx$

$= \int \{(\sin x)^{\frac{1}{2}} - \sin^{\frac{5}{2}} x\}(\sin x)'\,dx$

$= \frac{2}{3}(\sin x)^{\frac{3}{2}} - \frac{2}{7}(\sin x)^{\frac{7}{2}} + C$　（C＝積分定数）

$6\int \log x$

$= 6x \log x$

$6\int (x)' \log x - 6\frac{x}{x}$

$(\log x)^{2\prime}$

$= 3(\log x)^2 \times \frac{1}{x}$

(8) $\int (\log x)^3\,dx$

$x = e^t$ とおく

$\log x = t$

$\frac{dx}{dt} = e^t$

$dx = e^t dt$ より

$\int t^3 e^t\,dt$

$= \int (e^t)' t^3\,dt$

$= e^t t^3 - 3\int e^t t^2\,dt$ というように部分積分を続けて

$= t^3 e^t - 3t^2 e^t + 6t e^t - 6e^t + C$

$= x\{(\log x)^3 - 3(\log x)^2 + 6\log x - 6\} + C$　（C＝積分定数）

（別）$\int (x)'(\log x)^3\,dx$

$= x\log x^3 - 3\int (\log x)^2$

$(\log x)^{2\prime} = 2\frac{\log x}{x}$

$= x\log x^3 - 3\int (x)'(\log x)^2$

$= x(\log x)^3 - 3x(\log x)^2 + 3\int 2\log x\,dx$

$= x(\log x)^3 - 3x(\log x)^2 + 6x\log x - 6 + C$

(9) $\int e^x(x\cos x + \sin x)\,dx$

$= \int e^x x\cos x + \int e^x \sin x\,dx$

$I = \int e^x x\cos x$　$J = \int e^x \sin x$ とおくと

$I = e^x x\cos x - \int e^x(\cos x - x\sin x)\,dx + C$

$= e^x x\cos x - \int e^x \cos x + \int e^x x\sin x$

$= e^x x\cos x - e^x\cos x - \int e^x \sin x\,dx + e^x x\sin x - \int e^x(\sin x + x\cos x)\,dx + C$

$= e^x x\cos x - e^x\cos x + e^x x\sin x - 2J - I + C$

$\therefore I + J = \frac{1}{2}e^x x(\sin x + \cos x) - \frac{1}{2}e^x\cos x + C$

(10) $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}} = f(x)$

$x = \tan x$ とおくと

$dx = \frac{1}{\cos^2\theta}\,d\theta$

$f(x) = \int \frac{1}{(1+\tan^2\theta)^{\frac{3}{2}}} \cdot \frac{1}{\cos^2\theta}\,d\theta$

$= \int \cos\theta\,d\theta$

$= \sin\theta + C$

$= \sqrt{1 - \frac{1}{1+\tan^2\theta}} + C$

$= \sqrt{\frac{x^2}{1+x^2}} + C$