平成 23 年度　機械系の数学演習 I　 第 2 回
問題

※　うｐ主が手計算で解いたものです。どうせ間違いだらけだろうと思いますが、おかしなところがあれば、各自で修正してくださいな(´・ω・`)
明らかな間違いを発見された方は、wiki のコメント欄に書き込んでくだされば、皆さんのためになると思いますので、是非そうしてください。

1,　つぎの関数を微分せよ　($|x|<1$)：

$$\sqrt{\frac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}}$$

$y=\sqrt{\frac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}}$ とおくと、$y^2=\frac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}$

両辺を微分すると

$$2y\frac{dy}{dx}=\frac{\left\{\frac{d}{dx}\left(1-\sqrt[3]{x}\right)\right\}\left(1+\sqrt[3]{x}\right)-\left(1-\sqrt[3]{x}\right)\left\{\frac{d}{dx}\left(1+\sqrt[3]{x}\right)\right\}}{\left(1+\sqrt[3]{x}\right)^2}$$

$$2y\frac{dy}{dx}=\frac{\left(-\frac{1}{3}x^{-\frac{2}{3}}\right)\left(1+x^{\frac{1}{3}}\right)-\left(1-x^{\frac{1}{3}}\right)\left(\frac{1}{3}x^{-\frac{2}{3}}\right)}{\left(1+\sqrt[3]{x}\right)^2}$$

$$2y\frac{dy}{dx}=\frac{\left(-\frac{1}{3}x^{-\frac{2}{3}}\right)\left\{\left(1+x^{\frac{1}{3}}\right)+\left(1-x^{\frac{1}{3}}\right)\right\}}{\left(1+\sqrt[3]{x}\right)^2}$$

$$6y\frac{dy}{dx}=-\frac{2x^{-\frac{2}{3}}}{\left(1+\sqrt[3]{x}\right)^2}$$

$$\frac{dy}{dx}=-\frac{2x^{-\frac{2}{3}}}{6y\left(1+\sqrt[3]{x}\right)^2}$$

$$\frac{dy}{dx}=-\frac{x^{-\frac{2}{3}}}{3y\left(1+\sqrt[3]{x}\right)^2}$$

$$\frac{dy}{dx}=-\frac{x^{-\frac{2}{3}}}{3\left(\sqrt{\frac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}}\right)\left(1+\sqrt[3]{x}\right)^2}$$

$$\frac{dy}{dx}=-\frac{x^{-\frac{2}{3}}}{3\left(1+\sqrt[3]{x}\right)^{\frac{3}{2}}\sqrt{1-\sqrt[3]{x}}}$$

2, つぎの極限値を求めよ：

(1) $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$ (2) $\lim\limits_{x\to 0}\left(\cot x-\csc x\right)$

(1) $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$

< 公式：$\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ を用いる方法>

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)}$$

$$=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}$$

$$=\lim_{x\to 0}\left\{\left(\frac{\sin x}{x}\right)^2\frac{1}{1+\cos x}\right\}$$

$$=\frac{1}{1+\cos 0}$$

$$=\frac{1}{2}$$

<三角関数の半角公式：$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}$　を用いる方法>

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{\left(\frac{1-\cos x}{2}\right)}{\left(\frac{x^2}{2}\right)}$$

$$=\lim_{x\to 0}\frac{\left(\frac{1-\cos x}{2}\right)}{\left(\frac{x^2}{2}\right)}$$

$$=\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x^2}{2}\right)}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x^2}{4}\right)}$$

$$=\frac{1}{2}\lim_{x\to 0}\left\{\frac{\sin\frac{x}{2}}{\left(\frac{x}{2}\right)}\right\}^2$$

$$=\frac{1}{2}$$

<ロピタルの定理>

$\lim_{x\to 0}(\text{分子})=\lim_{x\to 0}(\text{分母})=0$ より、ロピタルの定理を用いると

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x^2)}$$

$$=\lim_{x\to 0}\frac{\sin x}{2x}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x}$$

$$=\frac{1}{2}$$

(2) $\lim_{x\to 0}(\cot x-\csc x)$

$$\lim_{x\to 0}(\cot x-\csc x)$$

$$=\lim_{x\to 0}\left(\frac{1}{\tan x}-\frac{1}{\sin x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\cos x}{\sin x}-\frac{1}{\sin x}\right)$$

$$=\lim_{x\to 0}\left(-\frac{1-\cos x}{\sin x}\right)$$

$$=\lim_{x\to 0}\left\{-\frac{(1-\cos x)(1+\cos x)}{(1+\cos x)\sin x}\right\}$$

$$=\lim_{x\to 0}\left\{-\frac{(1-\cos^2 x)}{(1+\cos x)\sin x}\right\}$$

$$=\lim_{x\to 0}\left\{-\frac{\sin^2 x}{(1+\cos x)\sin x}\right\}$$

$$=\lim_{x\to 0}\left(-\frac{\sin x}{1+\cos x}\right)$$
$$=-\frac{\sin 0}{1+\cos 0}$$
$$=0$$

3, つぎの関数を微分せよ （a,b は定数）:

$$\frac{\sin x}{\sqrt{a^2\cos^2 x+b^2\sin^2 x}}$$

$$\frac{d}{dx}\left(\frac{\sin x}{\sqrt{a^2\cos^2 x+b^2\sin^2 x}}\right)$$
$$=\frac{\frac{d}{dx}\left(\sqrt{a^2\cos^2 x+b^2\sin^2 x}\right)\sin x-\left(\sqrt{a^2\cos^2 x+b^2\sin^2 x}\right)\frac{d}{dx}(\sin x)}{a^2\cos^2 x+b^2\sin^2 x}$$
$$=\frac{\left\{\frac{1}{2}\cdot\frac{a^2(-2\cos x\sin x)+b^2(2\sin x\cos x)}{\sqrt{a^2\cos^2 x+b^2\sin^2 x}}\right\}\sin x-\left(\sqrt{a^2\cos^2 x+b^2\sin^2 x}\right)\cos x}{a^2\cos^2 x+b^2\sin^2 x}$$
$$=\frac{\left\{\frac{-2(a^2-b^2)\sin x\cos x}{2}\right\}\sin x-\left(a^2\cos^2 x+b^2\sin^2 x\right)\cos x}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$
$$=\frac{\left\{(b^2-a^2)\sin^2 x\cos x\right\}-\left(a^2\cos^3 x+b^2\sin^2 x\cos x\right)}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$
$$=\frac{\left(b^2\sin^2 x\cos x-a^2\sin^2 x\cos x\right)-\left(a^2\cos^3 x+b^2\sin^2 x\cos x\right)}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$
$$=\frac{\left\{\left(b^2\sin^2 x-a^2\sin^2 x\right)-\left(a^2\cos^2 x+b^2\sin^2 x\right)\right\}\cos x}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$

$$=\frac{\left(-a^2\sin^2 x-a^2\cos^2 x\right)\cos x}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$

$$=\frac{-a^2\left(\sin^2 x+\cos^2 x\right)\cos x}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$

$$=\frac{-a^2\left(\sin^2 x+\cos^2 x\right)\cos x}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$

$$=-\frac{a^2\cos x}{\left(a^2\cos^2 x+b^2\sin^2 x\right)^{\frac{3}{2}}}$$

4,　つぎの関数を微分せよ　：

(1) $\dfrac{e^{-x^2}}{\sqrt{x}}$　　　(2) $\log\left(x+\sqrt{x^2+1}\right)$

(1) $\dfrac{e^{-x^2}}{\sqrt{x}}$

$$\frac{d}{dx}\left(\frac{e^{-x^2}}{\sqrt{x}}\right)$$

$$=\frac{\frac{d}{dx}\left(e^{-x^2}\right)\sqrt{x}-e^{-x^2}\frac{d}{dx}\left(\sqrt{x}\right)}{x}$$

$$=\frac{\left\{e^{-x^2}\cdot(-2x)\right\}\sqrt{x}-e^{-x^2}\left(\frac{1}{2\sqrt{x}}\right)}{x}$$

$$=\frac{\left(-2x\sqrt{x}-\frac{1}{2\sqrt{x}}\right)\left(e^{-x^2}\right)}{x}$$

$$=\frac{\left\{\left(-2x\sqrt{x}\right)2\sqrt{x}-1\right\}e^{-x^2}}{2x\sqrt{x}}$$

$$=\frac{\left(-4x^2-1\right)e^{-x^2}}{2x\sqrt{x}}$$

$$=-\frac{\left(4x^2+1\right)e^{-x^2}}{2x\sqrt{x}}$$

(2) $\log\left(x+\sqrt{x^2+1}\right)$

$$\frac{d}{dx}\left\{\log\left(x+\sqrt{x^2+1}\right)\right\}$$

$$=\frac{\frac{d}{dx}\left(x+\sqrt{x^2+1}\right)}{x+\sqrt{x^2+1}}$$

$$=\frac{\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}$$

$$=\frac{\left(1+\frac{x}{\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}$$

$$=\frac{\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}$$

$$=\frac{\sqrt{x^2+1}+x}{\left(x+\sqrt{x^2+1}\right)\left(\sqrt{x^2+1}\right)}$$

$$=\frac{1}{\sqrt{x^2+1}}$$

5,　対数微分法によって、つぎの関数の導関数を求めよ：

$$\frac{1}{(x+3)^2(x+5)^2(x+7)}$$

$y=\dfrac{1}{(x+3)^2(x+5)^2(x+7)}$ とおき、両辺の絶対値をとると、

$$|y|=|x+3|^{-2}|x+5|^{-2}|x+7|^{-1}$$

これで両辺がともに正になるので、両辺の対数をとると、

$$\log|y|=\log\left(|x+3|^{-2}|x+5|^{-2}|x+7|^{-1}\right)$$

$$\log|y|=\log|x+3|^{-2}+\log|x+5|^{-2}+\log|x+7|^{-1}$$

$$\log|y|=-2\log|x+3|-2\log|x+5|-\log|x+7|$$

ここで、両辺を x 微分すると、

$$\frac{1}{y}\cdot\frac{dy}{dx}=-\frac{2}{x+3}-\frac{2}{x+5}-\frac{1}{x+7}$$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{-2(x+5)(x+7)-2(x+3)(x+7)-(x+3)(x+5)}{(x+3)(x+5)(x+7)}$$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{-2(x^2+12x+35)-2(x^2+10x+21)-(x^2+8x+15)}{(x+3)(x+5)(x+7)}$$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{(-2x^2-24x-70)+(-2x^2-20x-42)+(-x^2-8x-15)}{(x+3)(x+5)(x+7)}$$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{-5x^2-52x-127}{(x+3)(x+5)(x+7)}$$

$$\frac{dy}{dx}=\frac{-y\left(5x^2+52x+127\right)}{(x+3)(x+5)(x+7)}$$

$$\frac{dy}{dx}=-\frac{5x^2+52x+127}{(x+3)^3(x+5)^3(x+7)^2}$$

6,　つぎの極限値を求めよ：

(1) $\lim_{x\to 0}\dfrac{\sinh 2x}{x}$ (2) $\lim_{x\to 0}\dfrac{\cosh x-1}{x^2}$

(1) $\lim_{x\to 0}\dfrac{\sinh 2x}{x}$

＜ロピタルの定理＞

$$\lim_{x\to 0}\frac{\sinh 2x}{x}$$

$$=\lim_{x\to 0}\frac{\left(\dfrac{e^{2x}-e^{-2x}}{2}\right)}{x}$$

$$=\lim_{x\to 0}\frac{e^{2x}-e^{-2x}}{2x}$$

$\lim_{x\to 0}(\text{分子})=\lim_{x\to 0}(\text{分母})=0$ より、ロピタルの定理を用いると

$$=\lim_{x\to 0}\frac{\dfrac{d}{dx}\left(e^{2x}-e^{-2x}\right)}{\dfrac{d}{dx}(2x)}$$

$$=\lim_{x\to 0}\frac{2\left(e^{2x}+e^{-2x}\right)}{2}$$

$$=\lim_{x\to 0}\left(e^{2x}+e^{-2x}\right)$$

$$=e^0+e^0$$

$$=2$$

<公式：$\lim_{x\to 0}\dfrac{e^x-1}{x}=0$ を用いる方法>

$$\lim_{x\to 0}\frac{\sinh 2x}{x}$$

$$=\lim_{x\to 0}\frac{\left(\dfrac{e^{2x}-e^{-2x}}{2}\right)}{x}$$

$$=\lim_{x\to 0}\frac{e^{2x}-e^{-2x}}{2x}$$

$$=\lim_{x\to 0}\left(\frac{e^{2x}}{2x}-\frac{e^{-2x}}{2x}\right)$$

$$=\lim_{x\to 0}\left\{\left(\frac{e^{2x}-1}{2x}+\frac{1}{2x}\right)-\left(\frac{e^{-2x}-1}{2x}+\frac{1}{2x}\right)\right\}$$

$$=\lim_{x\to 0}\left\{\frac{e^{2x}-1}{2x}+\frac{e^{-2x}-1}{(-2x)}\right\}$$

$$=1+1$$

$$=2$$

(2) $\displaystyle\lim_{x\to 0}\frac{\cosh x-1}{x^2}$

$$\lim_{x\to 0}\frac{\cosh x-1}{x^2}$$

$$=\lim_{x\to 0}\frac{\left(\frac{e^{x}+e^{-x}}{2}\right)-1}{x^2}$$

$$=\lim_{x\to 0}\frac{e^{x}+e^{-x}-2}{2x^2}$$

$\displaystyle\lim_{x\to 0}($ 分子 $)=\lim_{x\to 0}($ 分母 $)=0$ より、ロピタルの定理を用いると

$$=\lim_{x\to 0}\frac{\frac{d}{dx}\left(e^{x}+e^{-x}-2\right)}{\frac{d}{dx}\left(2x^2\right)}$$

$$=\lim_{x\to 0}\frac{e^{x}-e^{-x}}{4x}$$

$\displaystyle\lim_{x\to 0}($ 分子 $)=\lim_{x\to 0}($ 分母 $)=0$ より、ロピタルの定理を用いると

$$=\lim_{x\to 0}\frac{\frac{d}{dx}\left(e^{x}-e^{-x}\right)}{\frac{d}{dx}(4x)}$$

$$=\lim_{x\to0}\frac{e^x+e^{-x}}{4}$$

$$=\frac{e^0+e^0}{4}$$

$$=\frac{1+1}{4}$$

$$=\frac{1}{2}$$

7, つぎの関数を微分せよ :

$$(1)\quad \sinh\frac{1}{x} \qquad\qquad (2)\quad \log\cosh x$$

(1) $\sinh\dfrac{1}{x}$

$$\frac{d}{dx}\left(\sinh\frac{1}{x}\right)$$

$$=\frac{d}{dx}\left(\frac{e^{\frac{1}{x}}-e^{-\frac{1}{x}}}{2}\right)$$

$$=\frac{e^{\frac{1}{x}}\frac{d}{dx}\left(\frac{1}{x}\right)-e^{-\frac{1}{x}}\frac{d}{dx}\left(-\frac{1}{x}\right)}{2}$$

$$=\frac{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)-e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)}{2}$$

$$=-\frac{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}{2x^2}$$

$$=-\frac{\left(\frac{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}{2}\right)}{x^2}$$

$$=-\frac{\cosh\frac{1}{x}}{x^2}$$

(2)　$\log\cosh x$

$$\frac{d}{dx}(\log\cosh x)$$

$$=\frac{\frac{d}{dx}(\cosh x)}{\cosh x}$$

$$=\frac{\frac{d}{dx}\left(\frac{e^x+e^{-x}}{2}\right)}{\cosh x}$$

$$=\frac{\left(\frac{e^x-e^{-x}}{2}\right)}{\cosh x}$$

$$=\frac{\sinh x}{\cosh x}$$

$$=\tanh x$$

8,　$y=\dfrac{\cos ecx}{\sinh\left(\frac{1}{x}\right)}$ はつぎの方程式をみたすことを証明せよ　：

$$\frac{dy}{dx}=y\left(-\cot x+\frac{1}{x^2}\coth\frac{1}{x}\right)$$

変形すると、$y=\dfrac{\cos ecx}{\sinh\left(\frac{1}{x}\right)}=\dfrac{1}{\sin x\sinh\left(\frac{1}{x}\right)}=\dfrac{2}{\left(e^{\frac{1}{x}}-e^{-\frac{1}{x}}\right)\sin x}$ であるから、

$$\frac{dy}{dx} = -2 \cdot \frac{\frac{d}{dx}\left\{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x\right\}}{\left\{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x\right\}^2}$$

$$= -2 \cdot \frac{\frac{d}{dx}\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x + \left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\frac{d}{dx}(\sin x)}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)^2 \sin^2 x}$$

$$= -2 \cdot \frac{\left\{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right) - e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)\right\}\sin x + \left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\cos x}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)^2 \sin^2 x}$$

$$= -2 \cdot \frac{\frac{1}{x^2}\left(-e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x + \left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\cos x}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)^2 \sin^2 x}$$

$$= \frac{2}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x} \cdot \frac{\frac{1}{x^2}\left(e^{\frac{1}{x}} + e^{-\frac{1}{x}}\right)\sin x - \left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\cos x}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x}$$

$$= y \cdot \frac{\frac{1}{x^2}\left(e^{\frac{1}{x}} + e^{-\frac{1}{x}}\right)\sin x - \left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\cos x}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x}$$

$$= y \cdot \left\{\frac{\frac{1}{x^2}\left(e^{\frac{1}{x}} + e^{-\frac{1}{x}}\right)\sin x}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x} - \frac{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\cos x}{\left(e^{\frac{1}{x}} - e^{-\frac{1}{x}}\right)\sin x}\right\}$$

$$= y \cdot \left(\frac{1}{x^2} \cdot \frac{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}{e^{\frac{1}{x}} - e^{-\frac{1}{x}}} - \frac{\cos x}{\sin x} \right)$$

$$= y \cdot \left\{ \frac{1}{x^2} \cdot \frac{\left(\dfrac{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}{2} \right)}{\left(\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{2} \right)} - \frac{\cos x}{\sin x} \right\}$$

$$= y \cdot \left(\frac{1}{x^2} \cdot \frac{\cosh \frac{1}{x}}{\sinh \frac{1}{x}} - \frac{\cos x}{\sin x} \right)$$

$$= y \cdot \left(\frac{1}{x^2} \cdot \frac{1}{\tanh \frac{1}{x}} - \frac{1}{\tan x} \right)$$

$$= y \cdot \left(\frac{1}{x^2} \cdot \coth \frac{1}{x} - \cot x \right)$$

9, つぎの関数を微分せよ：

$$\frac{x \arcsin x}{\sqrt{1 - x^2}} + \frac{1}{2} \log\left(1 - x^2\right)$$

$$\frac{d}{dx} \left\{ \frac{x \arcsin x}{\sqrt{1 - x^2}} + \frac{1}{2} \log\left(1 - x^2\right) \right\}$$

$$= \frac{d}{dx} \left(\frac{x \arcsin x}{\sqrt{1 - x^2}} \right) + \frac{1}{2} \cdot \frac{d}{dx} \left\{ \log\left(1 - x^2\right) \right\}$$

$$=\frac{\frac{d}{dx}(x\arcsin x)\sqrt{1-x^2}-x\arcsin x\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{1-x^2}+\frac{1}{2}\cdot\frac{\frac{d}{dx}(1-x^2)}{1-x^2}$$

$$=\frac{\left(\frac{x}{\sqrt{1-x^2}}+\arcsin x\right)\sqrt{1-x^2}-x\arcsin x\left(\frac{-x}{\sqrt{1-x^2}}\right)}{1-x^2}+\frac{1}{2}\cdot\frac{(-2x)}{1-x^2}$$

$$=\frac{\left(x+\sqrt{1-x^2}\arcsin x\right)+\left(\frac{x^2\arcsin x}{\sqrt{1-x^2}}\right)}{1-x^2}-\frac{x}{1-x^2}$$

$$=\frac{x}{1-x^2}+\frac{\sqrt{1-x^2}\arcsin x}{1-x^2}+\frac{\left(\frac{x^2\arcsin x}{\sqrt{1-x^2}}\right)}{1-x^2}-\frac{x}{1-x^2}$$

$$=\frac{\sqrt{1-x^2}\arcsin x}{1-x^2}+\frac{\left(\frac{x^2\arcsin x}{\sqrt{1-x^2}}\right)}{1-x^2}$$

$$=\frac{\arcsin x}{1-x^2}\cdot\left(\sqrt{1-x^2}+\frac{x^2}{\sqrt{1-x^2}}\right)$$

$$=\frac{\arcsin x}{1-x^2}\cdot\frac{1-x^2+x^2}{\sqrt{1-x^2}}$$

$$=\frac{\arcsin x}{(1-x^2)^{\frac{3}{2}}}$$

10, $y=(\arcsin x)^2$ はつぎの微分方程式をみたすことを証明せよ　：

(1) $\sqrt{1-x^2}\,y'=2\arcsin x$　　　(2) $(1-x^2)y''-xy'=2$

(1)

$$\frac{d}{dx}y = \frac{d}{dx}(\arcsin x)^2$$

$$y' = 2\arcsin x \cdot \frac{d}{dx}(\arcsin x)$$

$$y' = 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,y' = 2\arcsin x$$

(2)

$$\frac{d}{dx}\left(\sqrt{1-x^2}\,y'\right) = \frac{d}{dx}(2\arcsin x)$$

$$\left\{\frac{d}{dx}\left(\sqrt{1-x^2}\right)y' + \sqrt{1-x^2}\,\frac{d}{dx}y'\right\} = \frac{2}{\sqrt{1-x^2}}$$

$$\left(\frac{x}{\sqrt{1-x^2}}\,y' + \sqrt{1-x^2}\,y''\right) = \frac{2}{\sqrt{1-x^2}}$$

$$xy' + \left(1-x^2\right)y'' = 2$$