解答

$\boxed{2}$

(1) $y=\sqrt{\dfrac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}}$

$$y'=\frac{1}{2}\times\sqrt{\frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}}\times\left(\frac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}\right)'$$

$$=\frac{1}{2}\times\sqrt{\frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}}\times\frac{-\frac{1}{3}x^{-\frac{2}{3}}(1+\sqrt[3]{x})-(1-\sqrt[3]{x})\times\frac{1}{3}x^{-\frac{2}{3}}}{(1+\sqrt[3]{x})^2}$$

$$=\frac{1}{2}\times\sqrt{\frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}}\times-\frac{1}{3}x^{-\frac{2}{3}}\times\frac{2}{(1+\sqrt[3]{x})^2}$$

$$=\frac{1}{3\sqrt{1-\sqrt[3]{x^2}}\,(x+\sqrt[3]{x^2})}$$

(2) 1、 $\dfrac{1-\cos x}{x^2}=\dfrac{1-\cos^2 x}{x^2(1+\cos x)}$

$$=\frac{\sin^2 x}{x^2(1+\cos x)}$$

$$\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}=\frac{1}{2}$$

2. $\cot x-\mathrm{cosec}\,x$

$$=\frac{\cos x-1}{\sin x}$$

$$=\frac{\cos^2 x-1}{\sin x(\cos x+1)}$$

$$=\frac{-\sin^2 x}{\sin x(\cos x+1)}$$

$$=\frac{-\sin x}{\cos x+1}$$

$$\lim_{x\to 0}\frac{-\sin x}{\cos x+1}=0$$

No.　Date

(3) $f(x) = \dfrac{\sin x}{\sqrt{a^2\cos^2 x + b^2\sin^2 x}}$

$$f'(x) = \frac{\cos x\sqrt{a^2\cos^2 x + b^2\sin^2 x} - \sin x \cdot \dfrac{2a^2\cos x\cdot(-\sin x) + 2b^2\sin x\cos x}{2\sqrt{a^2\cos^2 x + b^2\sin^2 x}}}{a^2\cos^2 x + b^2\sin^2 x}$$

$$= \frac{\cos x(a^2\cos^2 x + b^2\sin^2 x) - \sin x(a^2\cos x\times(-\sin x) + b^2\sin x\cos x)}{(a^2\cos^2 x + b^2\sin^2 x)^{\frac{3}{2}}}$$

$$= \frac{a^2\cos x}{(a^2\cos^2 x + b^2\sin^2 x)^{\frac{3}{2}}}$$

(4) 1、 $f(x) = \dfrac{e^{-x^2}}{\sqrt{x}}$ とすると

$$f'(x) = \frac{(e^{-x^2})'\sqrt{x} - e^{-x^2}\times(\sqrt{x})'}{x}$$

$$= \frac{-2x\sqrt{x}\,e^{-x^2} - \dfrac{e^{-x^2}}{2\sqrt{x}}}{x}$$

$$= \frac{-e^{-x^2}(4x^2+1)}{2x\sqrt{x}}$$

2、 $g(x) = \log(x + \sqrt{x^2+1})$ とすると

$$g'(x) = \frac{(x+\sqrt{x^2+1})'}{x+\sqrt{x^2+1}}$$

$$= \frac{1 + \dfrac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

(5) $y = \dfrac{1}{(x+3)^2(x+5)^2(x+7)}$ として両辺で対数をとると

$\log y = \log\dfrac{1}{(x+3)^2(x+5)^2(x+7)}$ で両辺をxで微分して

$$\frac{y'}{y} = -\frac{2}{(x+3)} - \frac{2}{(x+5)} - \frac{1}{(x+7)}$$

$$\therefore\ y' = \frac{5x^2+52x+127}{(x+3)^3(x+5)^3(x+7)^2}$$

Date . . No.

(6) 1.
$$\lim_{x\to 0}\frac{\sinh 2x}{x}$$
$$=\lim_{x\to 0}\frac{2\sinh x\cosh x}{x}$$
$$=\lim_{x\to 0}\frac{e^{2x}-e^{-2x}}{2x}$$
$$=\lim_{x\to 0}\frac{e^{2x}-1}{2x}+\lim_{x\to 0}\frac{-(e^{-2x}-1)}{2x}=1+1=2\quad\left(\lim_{x\to 0}\frac{e^{x}-1}{x}=1\right)$$

2.
$$\lim_{x\to 0}\frac{\cosh x-1}{x^2}$$
$$=\lim_{x\to 0}\frac{e^{x}+e^{-x}-2}{2x^2}$$
$$=\lim_{x\to 0}\left(\frac{e^{\frac{x}{2}}+e^{-\frac{x}{2}}}{\frac{x}{2}}\right)^2\times\frac{1}{8}$$
(1)より $=2^2\times\frac{1}{8}=\frac{1}{2}$

(7) 1.
$$\left(\sinh\frac{1}{x}\right)'=\frac{1}{2}\left(e^{\frac{1}{x}}-e^{-\frac{1}{x}}\right)'$$
$$=\frac{1}{2}\left(-\frac{1}{x^2}e^{\frac{1}{x}}-\frac{1}{x^2}e^{-\frac{1}{x}}\right)$$
$$=-\frac{1}{2x^2}\left(e^{\frac{1}{x}}-e^{-\frac{1}{x}}\right)=-\frac{\cosh\frac{1}{x}}{x^2}$$

2.
$$(\log\cosh x)'=\left(\log\frac{e^{x}+e^{-x}}{2}\right)'$$
$$=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$
$$=\frac{\sinh x}{\cosh x}=\tanh x$$

(8) $y = \dfrac{1}{\sinh\frac{1}{x} \times \sin x}$

$(\sinh\frac{1}{x} \times \sin x)' = \cosh\frac{1}{x} \times (-\frac{1}{x^2}) \times \sin x + \sinh\frac{1}{x} \times \cos x$

$\dfrac{dy}{dx} = \dfrac{-(\sinh\frac{1}{x} \times \sin x)'}{(\sinh\frac{1}{x} \times \sin x)^2}$

$\dfrac{dy}{dx} = y\left(\dfrac{-\sinh\frac{1}{x}\cos x}{\sinh\frac{1}{x}\sin x} + \dfrac{\cosh\frac{1}{x} \times \frac{1}{x^2} \times \sin x}{\sinh\frac{1}{x}\sin x}\right)$

$\cancel{= y\left(-\cot x + \frac{1}{x^2}\coth x\right)}$

よって $y = \dfrac{\operatorname{cosec} x}{\sinh\frac{1}{x}}$ は $\dfrac{dy}{dx} = y\left(-\cot x + \frac{1}{x^2}\coth\frac{1}{x}\right)$ をみたす

(9) $y = \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}} + \dfrac{1}{2}\log(1-x^2)$

$y' = \dfrac{(x\sin^{-1}x)'\sqrt{1-x^2} - x\sin^{-1}x\,(1-x^2)^{\frac{1}{2}'}}{1-x^2} + \dfrac{1}{2} \times \dfrac{-2x}{1-x^2}$

$= \dfrac{\left(\sin^{-1}x + x\cdot\frac{1}{\sqrt{1-x^2}}\right)\sqrt{1-x^2} + x^2\sin^{-1}x \times \frac{1}{\sqrt{1-x^2}}}{1-x^2} - \dfrac{x}{1-x^2}$

$= \dfrac{\sqrt{1-x^2}\sin^{-1}x + x + x^2\frac{\sin^{-1}x}{\sqrt{1-x^2}} - x}{1-x^2}$

$= \dfrac{\sqrt{1-x^2}\sin^{-1}x + x^2\frac{\sin^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$

$= \dfrac{(1-x^2)\sin^{-1}x + x^2\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} = \dfrac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}$

(10) $y = (\sin^{-1}x)^2$

$y' = 2\sin^{-1}x \times \dfrac{1}{\sqrt{1-x^2}}$

$y'' = \dfrac{2}{1-x^2} + \dfrac{2x\sin^{-1}x}{(\sqrt{1-x^2})^3}$

(1) 左辺 $= \sqrt{1-x^2}\; 2\sin^{-1}x\; \dfrac{1}{\sqrt{1-x^2}}$

$= 2\sin^{-1}x =$ 右辺

(2) 左辺 $= (1-x^2)\left\{\dfrac{2}{1-x^2} + \dfrac{2x\sin^{-1}x}{(\sqrt{1-x^2})^3}\right\} - x\cdot 2\sin^{-1}x\,\dfrac{1}{\sqrt{1-x^2}}$

$= 2 =$ 右辺