

15th

$$\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \int_0^{\frac{\pi}{2}} \frac{2\cos^2 \frac{x}{2}}{4 + 5 \sin \frac{x}{2} \cos \frac{x}{2}} \cdot \frac{dx}{2\cos^2 \frac{x}{2}}$$

$$\tan \frac{x}{2} = t \quad \text{--- (1)}$$

$$\frac{dx}{2\cos^2 \frac{x}{2}} = dt$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\frac{2}{\cos^2 \frac{x}{2}} + 5 \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}} \cdot \frac{dx}{2\cos^2 \frac{x}{2}}$$

$$\begin{array}{l|l} x & 0 \rightarrow \frac{\pi}{2} \\ t & 0 \rightarrow 1 \end{array} \quad = \int_0^{\frac{\pi}{2}} \frac{1}{2(1+\tan^2 \frac{x}{2}) + 5\tan \frac{x}{2}} \cdot \frac{dx}{2\cos^2 \frac{x}{2}}$$

$$= \int_0^1 \frac{dt}{2t^2 + 5t + 2}$$

$$= \int_0^1 \frac{1}{3} \left(\frac{2}{2t+1} - \frac{1}{t+2} \right) dt$$

$$= \frac{1}{3} [\ln|2t+1| - \ln|t+2|]_0^1$$

$$= \frac{\ln 2}{3}$$

✓