$$\int_{r_0}^{r} d(mv) = \int_{r_0}^{r} \frac{mg}{\alpha} dr$$

$$mv\big|_r - mv_0\big|_{r_0} = \left[\frac{g\pi r^4}{3\alpha}\right]_{r_0}^{r}$$

$$\frac{4\pi}{3}(r^3 v - r_0^3 v_0) = \frac{g\pi}{3\alpha}(r^4 - r_0^4)$$

$$v = \frac{gr}{4\alpha} + \frac{r_0^3}{r^3}\left(v_0 - \frac{gr_0}{4\alpha}\right)$$

落下距離yは、

$$y = \int_0^t v\,dt = \frac{1}{\alpha}\int_{r_0}^{r} v\,dr$$

$$= \frac{gr_0^2}{8\alpha^2}\left(\frac{r^2}{r_0^2} + \frac{r_0^2}{r^2} - 2\right) + \frac{r_0 v_0}{2\alpha}\left(1 - \frac{r_0^2}{r^2}\right)$$

[5]

(1) $\frac{\partial F_x}{\partial y} = -ax^2$、$\frac{\partial F_y}{\partial x} = -ax^2$

$\therefore \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$、$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$、$\frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$ $(=0)$

より保存力である。

$$U = -\int_0^x F_x dx = \frac{1}{3}ax^3 y$$

(2) $\frac{\partial F_x}{\partial y} = -ax^2$、$\frac{\partial F_y}{\partial x} = 0$ $\therefore$ 保存力でない。

(3) $\frac{\partial F_x}{\partial y} = -axz = \frac{\partial F_y}{\partial x}$、

$\frac{\partial F_y}{\partial z} = -\frac{1}{2}ax^2 - y^2 = \frac{\partial F_z}{\partial y}$

$\frac{\partial F_z}{\partial x} = -axy = \frac{\partial F_x}{\partial z}$ $\therefore$ 保存力である。

$$U = -\int_0^z F_z dz = \frac{1}{2}ax^2yz + \frac{1}{3}y^3 z + \frac{1}{3}z^3$$

(4) $F_x = kx$、$F_y = ky$、$F_z = kz$

$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} = 0$、$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} = 0$、$\frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} = 0$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = k\sqrt{x^2+y^2+z^2} = kr$ $\therefore$ 保存力である

$$U = -\int_0^r F\,dr = -\frac{1}{2}kr^2$$

(5)

$F = -\frac{k}{r^2}e_r = -\frac{k}{r^3}r$ $(\because r e_r = r)$

$F_x = -\frac{k}{r^3}x$、$F_y = -\frac{k}{r^3}y$、$F_z = -\frac{k}{r^3}z$

$$\frac{\partial r}{\partial x} = \frac{d}{dx}\sqrt{x^2+y^2+z^2} = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$\frac{\partial r}{\partial y} = \frac{y}{r}$、$\frac{\partial r}{\partial z} = \frac{z}{r}$

$$\frac{\partial F_x}{\partial y} = -kx\frac{-3}{r^4}\frac{\partial r}{\partial y} = \frac{3kxy}{r^5}$$

$$\frac{\partial F_y}{\partial x} = -ky\frac{-3}{r^4}\frac{\partial r}{\partial x} = \frac{3kxy}{r^5}$$

$\therefore \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$

同様に、$\frac{\partial F_z}{\partial F_x} = \frac{\partial F_x}{\partial z}$、$\frac{\partial F_z}{\partial y} = \frac{\partial F_y}{\partial z}$

$\therefore$ 保存力である。

$$U = -\int F\,dr = \int \frac{k}{r^2}dr = -\frac{k}{r} + C$$

$U \to 0$ $(r \to \infty)$ から、$C = 0$

$$\therefore U = -\frac{k}{r}$$

[6] ポテンシャルを持つ条件

$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial F_x}$、$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$、$\frac{\partial F_z}{\partial x} = \frac{F_x}{\partial z}$

より、$a_{12} = a_{21}$、$a_{23} = a_{32}$、$a_{13} = a_{31}$

$F_x = -\frac{\partial U}{\partial x}$、$F_y = -\frac{\partial U}{\partial y}$、$F_z = -\frac{\partial U}{\partial z}$、$U=0$を満たすもの

$$U(x,y,z) = -\frac{a_{11}x^2}{2} - \frac{a_{22}y^2}{2} - \frac{a_{33}z^2}{2} - a_{12}xy - a_{23}yz - a_{31}zx$$

[7]

(1)(a)

$$W_{OPQ} = \int_0^a F_x dx \Big|_{y=0} + \int_0^b F_y dy \Big|_{x=a}$$

$$= \int_0^b a y^3 dy = \frac{1}{4} a y^4$$

$$W_{ORQ} = \int_0^b F_y dy \Big|_{y=0} + \int_0^a F_x dx \Big|_{y=b}$$

$$= \int_0^a b x^3 dx = \frac{1}{4} a^4 b$$

∴保存力でない。

(b)

$$W_{OPQ} = \int_0^a F_x dx \Big|_{y=0} + \int_0^b F_y dy \Big|_{x=a}$$

$$= \int_0^b a^2 y\, dy = \frac{1}{2} a^2 b^2$$

$$W_{ORQ} = \int_0^b F_y dy \Big|_{x=0} + \int_0^a F_x dx \Big|_{y=b}$$

$$= \int_0^a x b^2 dx = \frac{1}{2} a^2 b^2$$

∴保存力である

(2)(a) $\frac{\partial F_x}{\partial y} = x^3$、$\frac{\partial F_y}{\partial x} = 3x^2 y^2$

∴保存力でない

(b) $\frac{\partial F_x}{\partial y} = 2xy$, $\frac{\partial F_y}{\partial x} = 2xy$

∴保存力である。

$$U = -\int_0^x x y^2 dx = -\frac{1}{2} x^2 y^2 + C$$

初期条件より $C = 0$

$$\therefore U = -\frac{1}{2} x^2 y^2$$