問1、2 (1)、(2) テキスト類題より略。(3)もおまけみたいなもん。

(3) $r = 100t^2$、$\theta = t^3$ とすると、

(1)より、

$$\boldsymbol{v} = 200t\,\boldsymbol{e}_r + 100t^2 \cdot 3t^2\,\boldsymbol{e}_\theta$$

$t = 1$ のとき、

$$\boldsymbol{v} = 200\,\boldsymbol{e}_r + 300\,\boldsymbol{e}_\theta$$

から、
$$\begin{cases} \text{半径方向}; 200 \\ \text{周方向}; 300 \end{cases}$$

また、(2)より、

$$\boldsymbol{\alpha} = \left(200 - 100t^2 \cdot (3t^2)^2\right)\boldsymbol{e}_r + \left(\frac{1}{100t^2}\right)\frac{d}{dt}\left((100t^2)^2 \cdot 3t^2\right)\boldsymbol{e}_\theta$$

$$= (200 - 900t^6)\,\boldsymbol{e}_r + \frac{180000t^5}{100t^2}\,\boldsymbol{e}_\theta$$

$t = 1$ のとき、

$$\boldsymbol{\alpha} = -700\,\boldsymbol{e}_r + 1800\,\boldsymbol{e}_\theta$$

から、
$$\begin{cases} \text{半径方向}; -700 \\ \text{周方向}; 1800 \end{cases}$$

問3.

(1)

$$\mathbb{P}=\begin{pmatrix} r\omega t - r\sin\omega t \\ r - r\cos\omega t \end{pmatrix}$$

$$=\begin{pmatrix} r(\omega t-\sin\omega t) \\ r(1-\cos\omega t) \end{pmatrix}$$

(2)

$$\dot{\mathbb{P}}=\begin{pmatrix} r(\omega-\omega\cos\omega t) \\ r\omega\sin\omega t \end{pmatrix}$$

$$\therefore \begin{cases} v_x = r\omega(1-\cos\omega t) \\ v_y = r\omega\sin\omega t \end{cases}$$

$$v_0=\sqrt{v_x^2+v_y^2}=r\omega\sqrt{(1-\cos\omega t)^2+(\sin\omega t)^2}$$

$$=r\omega\sqrt{2(1-\cos\omega t)}$$

(3)

$$\ddot{\mathbb{P}}=\begin{pmatrix} r\omega^2\sin\omega t \\ r\omega^2\cos\omega t \end{pmatrix}$$

$$\therefore \begin{cases} a_x = r\omega^2\sin\omega t \\ a_y = r\omega^2\cos\omega t \end{cases}$$

$$a_0=\sqrt{a_x^2+a_y^2}=r\omega^2$$

(4) $r(1-\cos\omega t)=0$ となる $t>0$ のうち最小のものが t'.

$\therefore \cos\omega t=1$、$\omega t=0, 2\pi, \cdots$

$$\therefore t'=\frac{2\pi}{\omega}$$

このときの速度の大きさは、$r\omega\sqrt{2(1-\cos\omega\frac{2\pi}{\omega})}=0$

(5)

$$l=\int_0^{t'}\sqrt{v_x^2+v_y^2}\,dt=\int_0^{t'}r\omega\sqrt{2(1-\cos\omega t)}\,dt$$

$$=\sqrt{2}r\omega\int_0^{t'}\sqrt{2}\sin\frac{\omega t}{2}\,dt$$

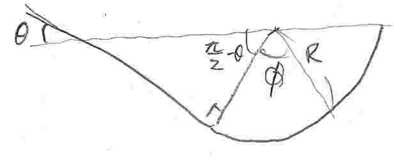
$$=2r\omega\left[-\frac{2}{\omega}\cos\frac{\omega t}{2}\right]_0^{t'}$$

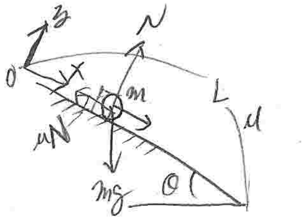
$$=-4r(-1-1)=8r$$

$$\left(\begin{aligned} \sin^2 x&=\frac{1-\cos 2x}{2} \\ \sin x&=\sqrt{\frac{1-\cos 2x}{2}} \\ \sqrt{2}\sin\frac{x}{2}&=\sqrt{1-\cos x} \end{aligned}\right)$$

↑意外にここでつまずく.
よくある変形なので覚

[解.





$\boldsymbol{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad m\ddot{\boldsymbol{r}} = \boldsymbol{F}$

$$m\ddot{\boldsymbol{r}} = m\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = N\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \mu N \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} mg\sin\theta \\ -mg\cos\theta \end{pmatrix}$$

$$\begin{cases} m\ddot{x} = -\mu N + mg\sin\theta \\ m\ddot{y} = N - mg\cos\theta \rightarrow 0 \end{cases}$$

$\ddot{y} = 0 \rightarrow N = mg\cos\theta$

$$m\ddot{x} = -\mu mg\cos\theta + mg\sin\theta = mg(\sin\theta - \mu\cos\theta)$$

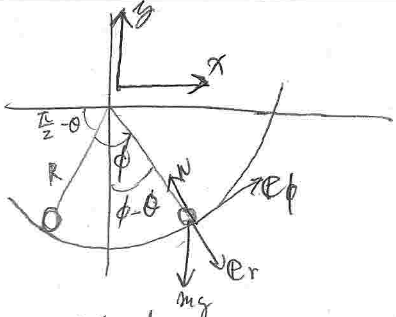
$$\dot{x} = (\sin\theta - \mu\cos\theta)gt + \underbrace{C}_{0}$$

$$x = (\sin\theta - \mu\cos\theta)\frac{g}{2}t^2$$

$x = L$ となる.

$$t = \sqrt{\frac{2L}{(\sin\theta - \mu\cos\theta)g}}$$

$$\therefore \dot{x} = \sqrt{2L(\sin\theta - \mu\cos\theta)g}$$



$$\boldsymbol{e}_r = \begin{pmatrix} \sin(\phi-\theta) \\ -\cos(\phi-\theta) \end{pmatrix}, \quad \boldsymbol{e}_\phi = \begin{pmatrix} \cos(\phi-\theta) \\ \sin(\phi-\theta) \end{pmatrix}$$

$$\boxed{\boldsymbol{r} = r\boldsymbol{e}_r}$$

$$\dot{\boldsymbol{r}} = \dot{r}\boldsymbol{e}_r + r\dot{\boldsymbol{e}}_r = \dot{r}\begin{pmatrix} \sin(\phi-\theta) \\ -\cos(\phi-\theta) \end{pmatrix} + r\begin{pmatrix} \dot{\phi}\cos(\phi-\theta) \\ \dot{\phi}\sin(\phi-\theta) \end{pmatrix} = \dot{r}\boldsymbol{e}_r + r\dot{\phi}\boldsymbol{e}_\phi$$

$$\ddot{\boldsymbol{r}} = \ddot{r}\boldsymbol{e}_r + 2\dot{r}\dot{\phi}\boldsymbol{e}_\phi + r\ddot{\phi}\boldsymbol{e}_\phi - r\dot{\phi}^2\boldsymbol{e}_r$$

$$m\ddot{\boldsymbol{r}} = (\ddot{r} - r\dot{\phi}^2)\boldsymbol{e}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi})\boldsymbol{e}_\phi$$

$$\| $$

$$\boldsymbol{F} = \left\{\begin{pmatrix} 0 \\ -mg \end{pmatrix}\cdot\boldsymbol{e}_r\right\}\boldsymbol{e}_r + \left\{\begin{pmatrix} 0 \\ -mg \end{pmatrix}\cdot\boldsymbol{e}_\phi\right\}\boldsymbol{e}_\phi - N\boldsymbol{e}_r$$

$$= \{mg\cos(\phi-\theta) - N\}\boldsymbol{e}_r - mg\sin(\phi-\theta)\boldsymbol{e}_\phi$$

$\boldsymbol{e}_r$: $m(\ddot{r} - r\dot{\phi}^2) = mg\cos(\phi-\theta) - N$

$\boldsymbol{e}_\phi$: $m(2\dot{r}\dot{\phi} + r\ddot{\phi}) = -mg\sin(\phi-\theta)$

また. $r = R$, $\dot{r} = 0$, $\ddot{r} = 0$

$$-mr\dot{\phi}^2 = mg\cos(\phi-\theta) - N$$

$$\therefore N = mg\cos(\phi-\theta) + \underbrace{mR\dot{\phi}^2}_{m\frac{v^2}{R}}$$

$\begin{pmatrix} v = m/s \\ R = m \\ \frac{v^2}{R} = m/s^2 \end{pmatrix}$

$$mR\ddot{\phi} = -mg\sin(\phi-\theta)$$

$$v = R\dot{\phi}$$

$$\ddot{\phi} = \frac{d^2\phi}{dt^2} = \frac{d\dot{\phi}}{dt} = \frac{d\phi}{dt}\frac{d\dot{\phi}}{d\phi} = \dot{\phi}\frac{d\left(\frac{v}{R}\right)}{d\phi} = \frac{v}{R^2}\frac{dv}{d\phi}$$

$$mR\ddot{\phi} = \underbrace{m\frac{v}{R}\frac{dv}{d\phi} = -mg\sin(\phi-\theta)}$$

$$\Downarrow$$

$$\int v\,dv = \int -Rg\sin(\phi-\theta)\,d\phi$$

$$\frac{1}{2}v^2 = Rg\cos(\phi-\theta) + C$$