

Homework #6

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1. Consider a free surface boundary with a normal vector $\mathbf{n} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$. What are the boundary conditions?

2. Consider a one-dimensional problem, i.e., displacement, density, and elastic properties are functions of z but not x or y . There is a free surface at $z = 0$, a water layer from $z = 0$ to $z = 1$ and a solid layer from $z = 1$ to $z = 2$. Write the weak form of the equation of motion, including the boundary term to ensure the necessary continuity conditions at the fluid-solid boundary.

3. Consider a P-SV problem with a boundary between two media at $z = 0$. The z -axis is positive upwards. The lower medium ($z \leq 0$) has density ρ_1 and isotropic elastic constants λ_1 and μ_1 . The upper medium ($z \geq 0$) has density ρ_2 and isotropic elastic constants λ_2 and μ_2 . Consider the case of an incoming P-wave from below with unit amplitude. In the first medium the displacement is

$$\mathbf{u} = (-k_x, 0, -k_{zP1}) \exp(i(-k_x x - k_{zP1} z)) + A_1(-k_x, 0, k_{zP1}) \exp(i(-k_x x + k_{zP1} z)) + B_1(k_{zS1}, 0, k_x) \exp(i(-k_x x + k_{zS1} z)) \quad (1)$$

and in the second medium the displacement is

$$\mathbf{u} = A_2(-k_x, 0, -k_{zP2}) \exp(i(-k_x x - k_{zP2} z)) + B_2(k_{zS2}, 0, -k_x) \exp(i(-k_x x - k_{zS2} z)). \quad (2)$$

Here A_1 , B_1 , A_2 and B_2 are the coefficients for the reflected P-wave, the reflected S-wave, transmitted P-wave, and transmitted S-wave, respectively. The dispersion equations are

$$k_x^2 + k_{zP1}^2 = \frac{\omega^2}{\alpha_1^2} \quad (3)$$

$$k_x^2 + k_{zP2}^2 = \frac{\omega^2}{\alpha_2^2} \quad (4)$$

$$k_x^2 + k_{zS1}^2 = \frac{\omega^2}{\beta_1^2} \quad (5)$$

$$k_x^2 + k_{zS2}^2 = \frac{\omega^2}{\beta_2^2} \quad (6)$$

(a) List any typographical errors in the problem that must be corrected.

(b) Write the equations for continuity of displacement and traction that can be used to determine A_1 , B_1 , A_2 and B_2 . Write these equations as simply as possible. You do not have to solve these equations.

Due: November 27, 2011 in class