

世界間隔は Lorentz 変換で不変

$$\begin{aligned}
 s'^2 &= (ct')^2 - x'^2 - y'^2 - z'^2 \\
 &= (\gamma ct - \gamma\beta x)^2 - (-\gamma\beta ct + \gamma x)^2 - y^2 - z^2 \\
 &= \gamma^2(1-\beta^2)(ct)^2 - \gamma^2(1-\beta^2)x^2 - y^2 - z^2 \\
 &= s^2
 \end{aligned}$$

回転から γ もの

$$\beta = \frac{v}{c} = \tanh \xi = \frac{e^\xi - e^{-\xi}}{e^\xi + e^{-\xi}} \quad \text{と置く}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh \xi \quad \cosh^2 \xi - \sinh^2 \xi = 1$$

$$\gamma\beta = \sinh \xi \quad \text{と置く}$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh \xi & -\sinh \xi & 0 & 0 \\ \sinh \xi & \cosh \xi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Lorentz 変換は一般に可換でない... non-Abelian

$$L_x(-\beta) = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad L_z(-\beta) = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

$$L_x(-\beta) L_z(-\beta) \neq L_z(-\beta) L_x(-\beta)$$

$L_x(-\beta_1) L_x(-\beta_2)$ - 同じ方向の Lorentz 変換は可換