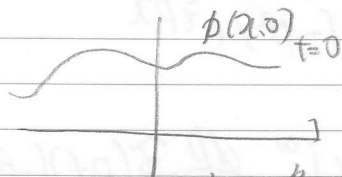


では初期条件が一般に $\phi(x, 0) = \rho(x)$ のときは?



Fourier 変換して $\widehat{\phi}(q, t) = C(q) e^{-vq^2 t}$ までは同じ
 $C(q)$ を決めることで、 $C(q) = \widehat{\phi}(q, 0) = \int_{-\infty}^{\infty} dx \rho(x) e^{-iqx} = \widehat{\rho}(q)$

よって $\widehat{\phi}(q, t) = \widehat{\rho}(q) e^{-vq^2 t}$

逆変換して得ると、

$$\begin{aligned} \phi(x, t) &= \int_{-\infty}^{\infty} \frac{dq}{2\pi} \widehat{\phi}(q, t) e^{iqx} = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \widehat{\rho}(q) e^{-vq^2 t} e^{iqx} \\ &= \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left(\int_{-\infty}^{\infty} dy \rho(y) e^{iqy} \right) e^{-vq^2 t} e^{iqx} \\ &= \int_{-\infty}^{\infty} dy \rho(y) \int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{-vq^2 t} e^{iq(x-y)} \end{aligned}$$

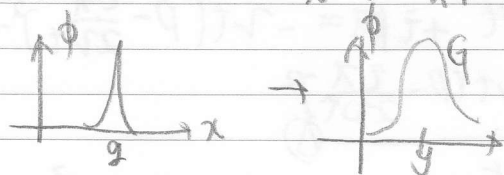
$$G(x-y, t) = \frac{1}{\sqrt{4\pi vt}} e^{-\frac{(x-y)^2}{4vt}}$$

$$\phi(x, t) = \int_{-\infty}^{\infty} dy \rho(y) G(x-y, t)$$

別の見方

$$G(x-y, t) = \frac{1}{\sqrt{4\pi vt}} e^{-\frac{(x-y)^2}{4vt}}$$

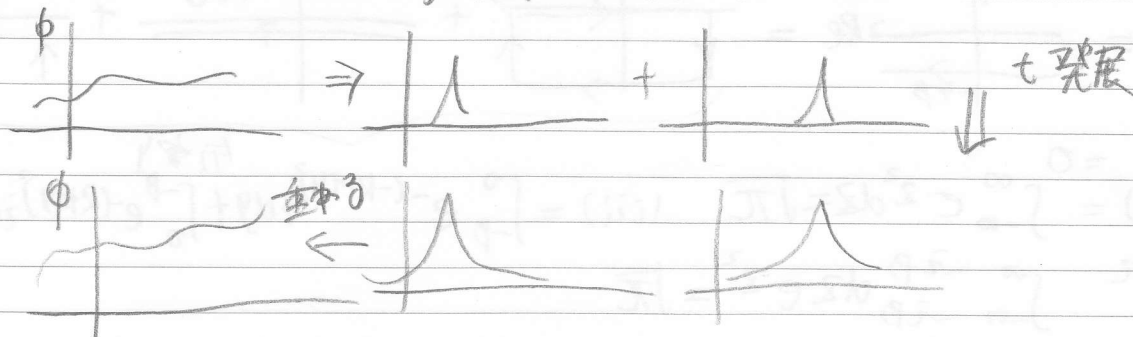
$\phi(x, 0) = \delta(x-y)$ は 1 行の解になっている



$$\begin{cases} \frac{\partial}{\partial t} G = v \frac{\partial^2}{\partial x^2} G \\ G(x-y, 0) = \delta(x-y) \end{cases}$$

初期条件の $\rho(x)$ を δ 関数の重ね合わせとすると

$$\phi(x, 0) = \rho(x) = \int dy \rho(y) \delta(x-y)$$



$$\phi(x, t) = \int dy \rho(y) G(x-y, t)$$

再 CHECK

$$\left(\frac{\partial}{\partial t} - v \frac{\partial^2}{\partial x^2} \right) \phi(x, t) = \int dy \rho(y) \left(\frac{\partial}{\partial t} - v \frac{\partial^2}{\partial x^2} \right) G(x-y, t) = 0$$

$$\phi(x, 0) = \int dy \rho(y) \frac{G(x-y, 0)}{\delta(x-y)} = \rho(x) \quad //$$