

P. 207 問 2 (1)

NO.

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$$\iint_{0 < x^2 + y^2 \leq M} \log(x^2 + y^2) dx dy$$

極座標で変換すると、積分領域は $\Omega = \{(r, \theta); 0 < r \leq \sqrt{M}, 0 \leq \theta < 2\pi\}$
 $\begin{pmatrix} x = r \cos \theta \\ y = r \sin \theta \end{pmatrix}$

$$J(r, \theta) = \begin{vmatrix} \frac{dx}{dr} & \frac{dx}{d\theta} \\ \frac{dy}{dr} & \frac{dy}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r \cos^2 \theta + r \sin^2 \theta = r$$

したがって、

$$(\text{与式}) = \iint_{\Omega} \log(r^2 \cos^2 \theta + r^2 \sin^2 \theta) \cdot J(r, \theta) dr d\theta$$

$$= \int_0^{2\pi} \left\{ \int_0^{\sqrt{M}} \log r^2 \cdot r dr \right\} d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{\sqrt{M}} 2r \log r dr$$

$$= 4\pi \lim_{\alpha \rightarrow 0} \int_{\alpha}^{\sqrt{M}} r \log r dr$$

$$= 4\pi \lim_{\alpha \rightarrow 0} \left\{ \left[\frac{r^2}{2} \log r \right]_{\alpha}^{\sqrt{M}} - \int_{\alpha}^{\sqrt{M}} \frac{r}{2} dr \right\}$$

$$= 2\pi \lim_{\alpha \rightarrow 0} \left\{ [r^2 \log r]_{\alpha}^{\sqrt{M}} - \left[\frac{r^2}{2} \right]_{\alpha}^{\sqrt{M}} \right\}$$

$$= \pi \lim_{\alpha \rightarrow 0} \left\{ 2(M \log \sqrt{M} - \alpha^2 \log \alpha) - (M - \alpha^2) \right\}$$

$$= \pi (M \log M - M)$$

$$= \pi M (\log M - 1)$$

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