

微分積分学Ⅱレポート課題 10/16 分

提出 20/2 10 23 火

P. 問4 (4) $\int_0^{\frac{\pi}{2}} \cos^m x \sin^n x dx$

(解) $I(m, n) = \int_0^{\frac{\pi}{2}} \cos^m x \sin^n x dx$ とする。

$m = 2k+1$ のとき、

$$\begin{aligned} I(2k+1, n) &= \int_0^{\frac{\pi}{2}} \cos^{2k+1} x \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^{2k} x \cdot \sin^n x (\sin x)' dx \\ &= \left[\frac{1}{n+1} \sin^{n+1} x \cos^{2k} x \right]_0^{\frac{\pi}{2}} - \frac{1}{n+1} \int_0^{\frac{\pi}{2}} (2k) \cos^{2k-1} x \cdot (-\sin x) \cdot \sin^{n+1} x dx \\ &= \frac{2k}{n+1} \int_0^{\frac{\pi}{2}} \cos^{2k-1} x \sin^{n+2} x dx = \frac{2k}{n+1} I(2k-1, n+2) = \frac{2k}{n+1} \cdot \frac{2k-2}{n+3} I(2k-3, n+4) \\ &= \dots = \frac{2k}{n+1} \cdot \frac{2k-2}{n+3} \cdot \dots \cdot \frac{2}{n+2k-1} I(1, n+2k) \end{aligned}$$

$$\therefore I(1, n+2k) = \int_0^{\frac{\pi}{2}} \cos x \sin^{n+2k} x dx = \int_0^{\frac{\pi}{2}} \sin^{n+2k} x (\sin x)' dx = \left[\frac{1}{n+2k+1} \sin^{n+2k+1} x \right]_0^{\frac{\pi}{2}} = \frac{1}{n+2k+1}$$

よって、

$$I(2k+1, n) = \frac{2k}{n+1} \cdot \frac{2k-2}{n+3} \cdot \dots \cdot \frac{2}{n+2k-1} \cdot \frac{1}{n+2k+1} = \frac{(2k)!!}{(n+1)(n+3) \dots (n+2k+1)}$$

$m = 2k$ のとき、

$$\begin{aligned} I(2k, n) &= \int_0^{\frac{\pi}{2}} \cos^{2k} x \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^{2k-1} x \sin^n x (\sin x)' dx \\ &= \left[\frac{1}{n+1} \sin^{n+1} x \cos^{2k-1} x \right]_0^{\frac{\pi}{2}} - \frac{1}{n+1} \int_0^{\frac{\pi}{2}} (2k-1) \cos^{2k-2} x \cdot (-\sin x) \cdot \sin^{n+1} x dx \\ &= \frac{2k-1}{n+1} \int_0^{\frac{\pi}{2}} \cos^{2k-2} x \sin^{n+2} x dx = \frac{2k-1}{n+1} I(2k-2, n+2) = \frac{2k-1}{n+1} \cdot \frac{2k-3}{n+3} I(2k-4, n+4) \\ &= \dots = \frac{2k-1}{n+1} \cdot \frac{2k-3}{n+3} \cdot \dots \cdot \frac{1}{n+2k-1} I(0, n+2k) \end{aligned}$$

$$\therefore I(0, n+2k) = \int_0^{\frac{\pi}{2}} \sin^{n+2k} x dx = I_{2n+2k} \quad (P. \text{ の記号 })$$

よって、

$$I(2k, n) = \frac{2k-1}{n+1} \cdot \frac{2k-3}{n+3} \cdot \dots \cdot \frac{1}{n+2k-1} I_{2n+2k} = \frac{(2k-1)!!}{(n+1)(n+3) \dots (n+2k-1)} I_{2n+2k}$$