(ii). $\lim_{x \to 0} \frac{x^2 \sin \frac{1}{x}}{\sin x} = \lim_{x \to 0} \frac{x}{\sin x} \times \frac{\sin \frac{1}{x}}{\frac{1}{x}}$

$= 1 \times 1 = \underline{1}$ //

問題4.

(i) $(\sin x)' = \cos x$. $(\sin x)'' = -\sin x$

$(\sin x)''' = -\cos x$ $(\sin x)^{(4)} = \sin x$ より.

$\sin x = \sin 0 + x \cos 0 + \frac{1}{2!} x^2 (-\sin 0) + \cdots$

$= x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \cdots$

$= \underline{\sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^{2k-1}}{(2k-1)!}}$ // より.

$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \lim_{x \to 0} \frac{x - x + \frac{1}{3!}x^3 - \sum_{k=3}^{\infty} \frac{(-1)^{k-1} x^{2k-1}}{(2k-1)!}}{x^3}$

$\lim_{x \to 0} \frac{\sum_{k=3}^{\infty} \frac{(-1)^{k-1} x^{2k-1}}{(2k-1)!}}{x^3} = 0$ より.

$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \frac{1}{3!} = \underline{\frac{1}{6}}$ //

(ii). $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right) = \lim_{x \to 0} \frac{1}{x^2} \left(1 - \frac{x^2}{\sin^2 x} \right)$

$= \lim_{x \to 0} \frac{1}{x^2} \left(1 - \frac{x}{\sin x} \right) \left(1 + \frac{x}{\sin x} \right)$.

$\lim_{x \to 0} \left(1 + \frac{x}{\sin x} \right) = 1 + 1 = 2$ より. $\lim_{x \to 0} \frac{1}{x^2} \left(1 - \frac{x}{\sin x} \right)$

を考える.

$\frac{1}{x^2} \left(1 - \frac{1}{\sin^2 x} \right) = \frac{x}{\sin x} \times \frac{\sin x - x}{x^3}$ より (i) から.

$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right) = \lim_{x \to 0} \frac{x}{\sin x} \times (-1) \times \frac{x - \sin x}{x^3} \times \left(1 + \frac{x}{\sin x} \right)$

$= 1 \times (-1) \times \frac{1}{6} \times 2 = \underline{-\frac{1}{3}}$ //