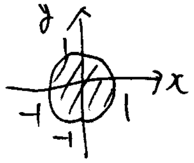
[5] $\displaystyle\int_{x^2+y^2\leq 1}(x^2+y^2)\,dx\,dy$ $\quad \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$



$\rightarrow \begin{cases} 0\leq r\leq 1 \\ 0\leq\theta\leq 2\pi \end{cases}$

$J(r,\theta) = r$

$$= \int_{r=0}^{r=1}\left\{\int_{\theta=0}^{\theta=2\pi}(r^2\cos^2\theta + r^2\sin^2\theta)\cdot r\,d\theta\right\}dr$$

$$= \int_{r=0}^{r=1}\left(\int_{\theta=0}^{\theta=2\pi} r^3\,d\theta\right)dr = \int_0^{2\pi}d\theta\cdot\int_0^1 r^3\,dr = 2\pi\left[\frac{r^4}{4}\right]_0^1$$

$$= 2\pi\cdot\frac{1}{4} = \frac{\pi}{2}$$

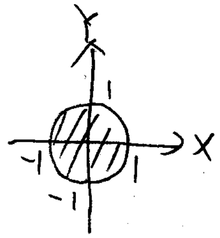
[6] $x-y = r\cos\theta$ $\quad x = \dfrac{r(\cos\theta+\sin\theta)}{2}$

$x+y = r\sin\theta$ $\quad y = \dfrac{r(\sin\theta-\cos\theta)}{2}$

$x-y = X$

$x+y = Y$

$X^2+Y^2\leq 1$



$\rightarrow \begin{cases} 0\leq r\leq 1 \\ 0\leq\theta\leq 2\pi \end{cases}$

$$J(r,\theta) = \begin{vmatrix} \dfrac{\cos\theta+\sin\theta}{2} & \dfrac{r(-\sin\theta+\cos\theta)}{2} \\ \dfrac{\sin\theta-\cos\theta}{2} & \dfrac{r(\cos\theta+\sin\theta)}{2} \end{vmatrix}$$

$$= \frac{r}{4}(\cos\theta+\sin\theta)^2 + \frac{r}{4}(\sin\theta-\cos\theta)^2$$

$$= \frac{r}{4}(2\cos^2\theta + 2\sin^2\theta) = \frac{r}{2}$$

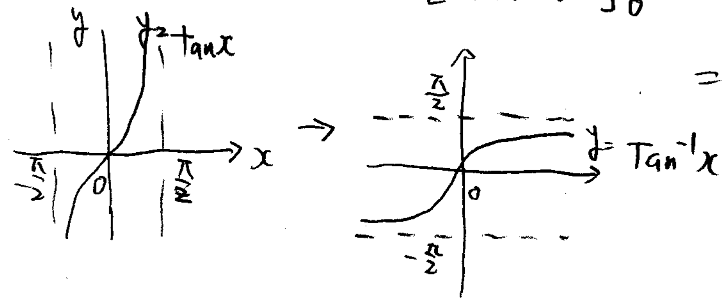
$$\int_{(x-y)^2+(x+y)^2\leq 1}dx\,dy = \int_{\theta=0}^{\theta=2\pi}\left\{\int_{r=0}^{r=1}\frac{r}{2}\,dr\right\}d\theta$$

$$= 2\pi\cdot\frac{1}{2}\int_0^1 r\,dr = \pi\left[\frac{r^2}{2}\right]_0^1 = \frac{\pi}{2}$$

問題3 [1] $\displaystyle\int_0^\infty e^{-x}\,dx = \left[-e^{-x}\right]_0^\infty = -\frac{1}{e^\infty} + e^0 = 1$

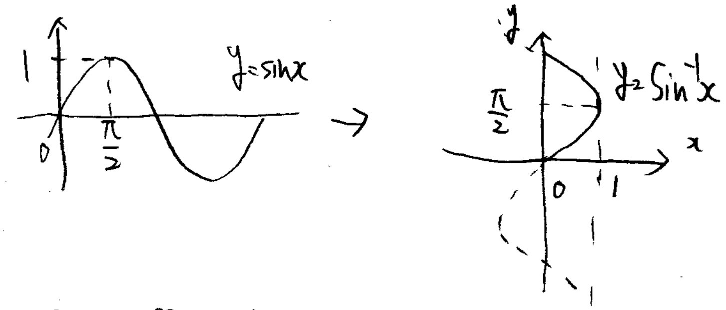
[2] $\displaystyle\int_0^\infty\frac{1}{x^2+1}\,dx = \left[\operatorname{Tan}^{-1}x\right]_0^\infty = \operatorname{Tan}^{-1}\infty - \operatorname{Tan}^{-1}0$

$$= \frac{\pi}{2}$$



[3] $\displaystyle\int_0^1\frac{1}{\sqrt{1-x^2}}\,dx = \left[\operatorname{Sin}^{-1}x\right]_0^1 = \operatorname{Sin}^{-1}1 - \operatorname{Sin}^{-1}0$

$$= \frac{\pi}{2}$$



[4] $\displaystyle\int_{-\infty}^\infty\frac{1}{e^x+e^{-x}}\,dx$ $\qquad e^x = t$ とおく $\quad dt = e^x dx = t\,dx$

$$= \int_0^\infty\frac{1}{t+\frac{1}{t}}\cdot\frac{dt}{t} = \int_0^\infty\frac{dt}{t^2+1}$$

x	$-\infty \to \infty$
t	$0 \to \infty$

$$= \left[\operatorname{Tan}^{-1}t\right]_0^\infty = \frac{\pi}{2}$$

[5] $\displaystyle\int_{-\infty}^\infty e^{-x^2}dx\int_{-\infty}^\infty e^{-y^2}dy = \int_{\mathbb{R}^2}e^{-(x^2+y^2)}\,dx\,dy$

$\left(\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow \mathbb{R}^2 \rightarrow \begin{cases} 0\leq r<\infty \\ 0\leq\theta\leq 2\pi \end{cases},\ J(r,\theta) = r\right)$

$$= \int_{r=0}^{r=\infty}\left\{\int_{\theta=0}^{\theta=2\pi}e^{-r^2}\cdot r\,d\theta\right\}dr = \int_0^{2\pi}d\theta\int_0^\infty re^{-r^2}\,dr$$

$$= 2\pi\int_0^\infty\frac{1}{2}(-e^{-r^2})'\,dr = \pi\left[-e^{-r^2}\right]_0^\infty = \pi\left(-\frac{1}{e^\infty}+e^0\right) = \pi$$