問題1 [1] $\int \frac{dx}{x^2+1} = \mathrm{Tan}^{-1} x$

[2] $\int \frac{dx}{x+1} = \log(x+1)$

[3] $\int \frac{2x}{x^2+1} dx = \int \frac{(x^2+1)'}{x^2+1} dx$

$= \log(x^2+1)$

[4] $\int \frac{dx}{x^2-x} = \int \left(\frac{1}{x-1} - \frac{1}{x} \right) dx$

$= \log(x-1) - \log x$

[5] $\int \frac{2x-1}{x^2-x} dx = \int \frac{(x^2-x)'}{x^2-x} dx$

$= \log(x^2-x)$

[6] $\int \frac{x}{x-1} dx = \int \left(1 + \frac{1}{x-1} \right) dx$

$= x + \log(x-1)$

[7] $\int \frac{2}{x^2-1} dx = \int \left(\frac{1}{x-1} - \frac{1}{x+1} \right) dx$

$= \log(x-1) - \log(x+1)$

[8] $\int \frac{x^2+2x-1}{x^3-x^2+x-1} dx = \int \frac{x^2+2x-1}{(x-1)(x^2+1)} dx$

$= \int \left(\frac{1}{x-1} + \frac{2}{x^2+1} \right) dx = \log(x-1) + 2\,\mathrm{Tan}^{-1} x$

問題2 [1] $\iint_{\substack{0 \le x \le 2 \\ 0 \le y \le 1}} dx\,dy = \int_{x=0}^{x=2} \left\{ \int_{y=0}^{y=1} dy \right\} dx = \int_{x=0}^{x=2} [y]_{y=0}^{y=1} dx$

$= \int_0^2 dx = 2$

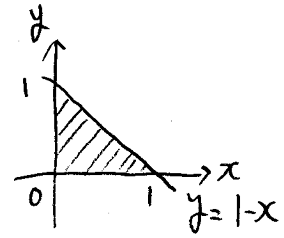
[2] $\iint_{\substack{0 \le x \le \pi \\ 0 \le y \le \frac{\pi}{2}}} \sin(x-y)\,dx\,dy = \int_{x=0}^{x=\pi} \left\{ \int_{y=0}^{y=\frac{\pi}{2}} \sin(x-y)\,dy \right\} dx$

$= \int_{x=0}^{x=\pi} \left[\cos(x-y) \right]_{y=0}^{y=\frac{\pi}{2}} dx = \int_0^{\pi} \left\{ \cos\left(x-\frac{\pi}{2}\right) - \cos x \right\} dx$

$= \left[\sin\left(x - \frac{\pi}{2}\right) - \sin x \right]_0^{\pi} = \sin\frac{\pi}{2} - \sin\pi - \sin\left(-\frac{\pi}{2}\right) + \sin 0$

$= 1 + 1 = 2$

[3] $\iint_{\substack{x \ge 0,\, y \ge 0 \\ x+y \le 1}} dx\,dy$



$0 \le x \le 1$
$0 \le y \le 1-x$

$= \int_{x=0}^{x=1} \left\{ \int_{y=0}^{y=1-x} dy \right\} dx = \int_{x=0}^{x=1} [y]_{y=0}^{y=1-x} dx$

$= \int_0^1 (1-x)\,dx = \left[x - \frac{1}{2}x^2 \right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$

[4] $\iint_{\substack{x \ge 0,\, y \ge 0 \\ x+y \le 1}} e^{x+y}\,dx\,dy = \int_{x=0}^{x=1} \left\{ \int_{y=0}^{y=1-x} e^{x+y}\,dy \right\} dx$

$= \int_{x=0}^{x=1} e^x \left[e^y \right]_{y=0}^{y=1-x} dx = \int_0^1 e^x \left\{ e^{(1-x)} - 1 \right\} dx$

$= \int_0^1 (e - e^x)\,dx = \left[ex - e^x \right]_0^1 = e - e - (0 - e^0)$

$= 1$