2009 後半 ?

微分積分学I　過去問

NO. ②

DATE

[2] 次の関数の( )内で定義された偏導関数(導関数)を求めよ。

(1) $z = e^{x^2+y^2}$　(z_x, z_y)

(解) $z_x = 2x\,e^{x^2+y^2}$、$z_y = 2y\,e^{x^2+y^2}$

(2) $z = \tan^{-1}\left(\frac{y}{x}\right)$　(z_x, z_y)
(おそらく tan の逆関数 Tan^{-1} だと思われる)

(解)
$$z_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2}$$

$$z_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} = \frac{1}{x+\frac{y^2}{x}} = \frac{x}{x^2+y^2}$$

(3) $u = e^x \sin y$　(u_{xx}, u_{yy}, u_{xy})

(解) $u_x = e^x \sin y$、$u_{xx} = e^x \sin y$、$u_{xy} = e^x \cos y$

$u_y = e^x \cos y$、$u_{yy} = -e^x \sin y$　$(u_{yx} = e^x \cos y)$

(4) $x^3 - 3axy + y^3 = 0$　$\left(\frac{dy}{dx}\right)$

(解) 両辺を x で微分すると、

$$\cancel{3}x^2 - \left(\cancel{3}ay + \cancel{3}ax\cdot\frac{dy}{dx}\right) + \cancel{3}y^2\cdot\frac{dy}{dx} = 0$$

$$(y^2 - ax)\frac{dy}{dx} + x^2 - ay = 0 \qquad \therefore\ \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

(5) $z = \sin^{-1}(xy)$、$x = \sin(u+v)$、$y = \sin(u-v)$　(z_u, z_v)
(これも sin の逆関数 Sin^{-1})

(解) $\frac{\partial x}{\partial u} = \cos(u+v)$、$\frac{\partial x}{\partial v} = \cos(u+v)$、$\frac{\partial y}{\partial u} = \cos(u-v)$、$\frac{\partial y}{\partial v} = -\cos(u-v)$

$$z_u = \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u} = \frac{y}{\sqrt{1-x^2y^2}}\cdot\cos(u+v) + \frac{x}{\sqrt{1-x^2y^2}}\cdot\cos(u-v)$$

$$= \frac{1}{\sqrt{1-x^2y^2}}\{y\cos(u+v) + x\cos(u-v)\}$$

$$z_v = \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v} = \frac{y}{\sqrt{1-x^2y^2}}\cdot\cos(u+v) + \frac{x}{\sqrt{1-x^2y^2}}\{-\cos(u-v)\}$$

$$= \frac{1}{\sqrt{1-x^2y^2}}\{y\cos(u+v) - x\cos(u-v)\}$$