問題 1. (あ) 収束する.

問題 2.

(1) $\displaystyle\lim_{x\to+\infty}\frac{\log x}{x}=\lim_{x\to+\infty}\frac{\frac{1}{x}}{1}=\lim_{x\to+\infty}\frac{1}{x}=0.$

(2) $\displaystyle\lim_{x\to1}\frac{\mathrm{Sin}^{-1}x}{\tanh^{-1}x}=0.$

(3) $\displaystyle\lim_{x\to0}\frac{\sin(x^2+2x)}{x^2+x}=\lim_{x\to0}\frac{(2x+2)\cos(x^2+2x)}{2x+1}=2.$

(4) $\displaystyle\lim_{x\to0}\frac{\mathrm{Tan}^{-1}x}{x}=\lim_{x\to0}\frac{\frac{1}{x^2+1}}{1}=1.$

問題 3.

(1) $f(x)=-x+2x^2$ (2) $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n}x^n$

問題 4.

(1) $\displaystyle\frac{\partial^2 f}{\partial x^2}=6x-2y,\quad \frac{\partial^2 f}{\partial x\partial y}=-2x+2y,\quad \frac{\partial^2 f}{\partial y^2}=2x-6y.$

(2) $\displaystyle\frac{\partial^2 f}{\partial x^2}=\frac{1}{x+y},\quad \frac{\partial^2 f}{\partial x\partial y}=\frac{1}{x+y},\quad \frac{\partial^2 f}{\partial y^2}=\frac{1}{x+y}.$

(3) $\displaystyle\frac{\partial^2 f}{\partial x^2}=y^2\cosh(xy),\quad \frac{\partial^2 f}{\partial x\partial y}=\sinh(xy)+xy\cosh(xy),\quad \frac{\partial^2 f}{\partial y^2}=x^2\cosh(xy).$

(4) $\displaystyle\frac{\partial^2 f}{\partial x^2}=\frac{3x^2-y^2}{(x^2+y^2)^3},\quad \frac{\partial^2 f}{\partial x\partial y}=\frac{4xy}{(x^2+y^2)^3},\quad \frac{\partial^2 f}{\partial y^2}=\frac{3y^2-x^2}{(x^2+y^2)^3}.$

問題 5.

(1) $\displaystyle(\frac{1}{n}\sum_{i=1}^{n}a_i,\frac{1}{n}\sum_{i=1}^{n}b_i).$ (2) (い) 極小である.