

2009年度 微分積分学Ⅰ 期末試験 (玉木)

①

1.

ロピタルの定理より、不定形にならず、

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin(x^2)}{1 - \cos^2 x} &= \lim_{x \rightarrow 0} \left\{ \frac{\sin(x^2)}{(\sin^2 x)} \right\}' = \lim_{x \rightarrow 0} \frac{2x \cos x^2}{2 \sin x \cos x} \\ &= \lim_{x \rightarrow 0} \frac{x}{\sin x} \times \cos x \\ &= 1\end{aligned}$$

(別解)

$$\begin{aligned}\frac{\sin(x^2)}{1 - \cos^2 x} &= \frac{\sin(x^2)}{\sin^2 x} = \frac{\sin(x)}{x^2} \times \left(\frac{x}{\sin x} \right)^2 \\ &= 1 \quad \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right)\end{aligned}$$

2.

$$f(x, y) = \sinh(xy) = \frac{e^{xy} - e^{-xy}}{2}$$

$$\begin{aligned}\frac{\partial f(x, y)}{\partial x} &= \frac{ye^{xy} + ye^{-xy}}{2} = y \left(\frac{e^{xy} + e^{-xy}}{2} \right) \\ &= y \cosh(xy) \quad \parallel\end{aligned}$$

yについても同様に、

$$\begin{aligned}\frac{\partial f(x, y)}{\partial y} &= \frac{xe^{xy} + xe^{-xy}}{2} = x \left(\frac{e^{xy} + e^{-xy}}{2} \right) \\ &= x \cosh(xy) \quad \parallel\end{aligned}$$

3.

$$g(x, y) = \sin^{-1} \sqrt{(x^2 + y^2)}$$

$$\begin{aligned}\frac{\partial g(x, y)}{\partial x} &= \frac{1}{\sqrt{1 - (x^2 + y^2)}} \times \left\{ \sqrt{(x^2 + y^2)} \right\}' \\ &= \frac{1}{\sqrt{1 - x^2 - y^2}} \times \frac{2x}{\sqrt{x^2 + y^2}} \times \frac{1}{2} \\ &= \frac{x}{\sqrt{(1 - x^2 - y^2)(x^2 + y^2)}} \quad \parallel\end{aligned}$$

yについても同様に、

$$\frac{\partial g(x, y)}{\partial y} = \frac{y}{\sqrt{(1 - x^2 - y^2)(x^2 + y^2)}} \quad \parallel$$