$$\therefore \quad \Delta U - \Delta T = \left(1 - \frac{m_1+m_2}{m_1+m_2+\frac{I}{r^2}}\right)(m_1-m_2)gh$$

$$\simeq \frac{m_1-m_2}{m_1+m_2+\frac{I}{r^2}} gh\frac{I}{r^2} \underset{\uparrow}{=} \ddot{z}_1 h \frac{I}{r^2}$$

(2)

$\dot{z}_1^2 = 2\ddot{z}_1 h$ だから $\ddot{z}_1 h = \frac{1}{2}\dot{z}_1^2 \underset{\uparrow}{=} \frac{1}{2} r^2\dot{\varphi}$

$\dot{z}_1 = r\dot{\varphi}$

$$\therefore \quad \Delta U - \Delta T = \frac{1}{2} I \dot{\varphi}^2 \quad // \quad ⑤$$

問2 (1) $T = \frac{m}{2}(v_r^2 + v_\theta^2) = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2)$ // ⑩

(2) $\frac{dW}{dt} = m(a_r v_r + a_\theta v_\theta) = m\{\dot{r}(\ddot{r} - r\dot{\theta}^2) + r\dot{\theta}(2\dot{r}\dot{\theta} + r\ddot{\theta})\}$

$= m(\dot{r}\ddot{r} + r\dot{r}\dot{\theta}^2 + r^2\dot{\theta}\ddot{\theta})$

$-\frac{1}{2}$(1)より $\frac{dT}{dt} = m(\dot{r}\ddot{r} + r\dot{r}\dot{\theta}^2 + r^2\dot{\theta}\ddot{\theta})$ これは上式と同じ

$$\therefore \quad \frac{dW}{dt} = \frac{dT}{dt} \quad // \quad ⑩$$

(3) Fは中心力だから $F_\theta = 0$

$$\therefore \quad \frac{dW}{dt} = F_r v_r = -\frac{GmM}{r^2}\dot{r} = \frac{d}{dt}\left(\frac{GmM}{r}\right) = -\frac{dU}{dt}$$

(2)より 左辺は $\frac{dT}{dt}$ に等しいから

$$\frac{dT}{dt} + \frac{dU}{dt} = \frac{d}{dt}(T+U) = 0 \quad \therefore E = T+U \text{は一定} \quad //$$

⑤