

$$\begin{aligned}
 (1) \quad \mathbf{v}(t) &= \frac{d}{dt} \mathbf{r}(t) \\
 &= \frac{d}{dt} (a \cos \omega t \cdot \hat{i} + a \sin \omega t \cdot \hat{j}) \\
 &= \underline{-(\omega a \sin \omega t) \hat{i} + (\omega a \cos \omega t) \hat{j}}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \mathbf{v}(t) \cdot \mathbf{r}(t) &= (-\omega a \sin \omega t) \times a \cos \omega t + (\omega a \cos \omega t) \times a \sin \omega t \\
 &= 0 \\
 \therefore \mathbf{v}(t) &\perp \mathbf{r}(t)
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad \mathbf{a}(t) &= \frac{d}{dt} \mathbf{v}(t) \\
 &= \underline{-(\omega^2 a \cos \omega t) \hat{i} - (\omega^2 a \sin \omega t) \hat{j}} \\
 \mathbf{a}(t) &= -\omega^2 \mathbf{r}(t) \quad \text{if } \mathbf{a}(t) \text{ is } \mathbf{r}(t) \text{ 反平行}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad m \frac{d^2 \mathbf{r}}{dt^2} &= -m \omega^2 \mathbf{r} \quad (\because (3)) \\
 &= -\frac{m \omega^2}{a^2} \mathbf{r} \quad (\because \omega = \frac{v}{a})
 \end{aligned}$$

(5) (4) について

$$m \frac{d\mathbf{v}}{dt} + m \omega^2 \mathbf{r} = 0$$

$$\text{両辺に } \mathbf{v} (= \frac{d\mathbf{r}}{dt}) \text{ をかけると}$$

$$m \mathbf{v} \cdot \frac{d\mathbf{v}}{dt} + m \omega^2 \mathbf{r} \cdot \frac{d\mathbf{r}}{dt} = 0$$

$$\therefore \frac{d}{dt} \left(\frac{1}{2} m \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} m \omega^2 \mathbf{r} \cdot \mathbf{r} \right) = 0$$

よって E は一定