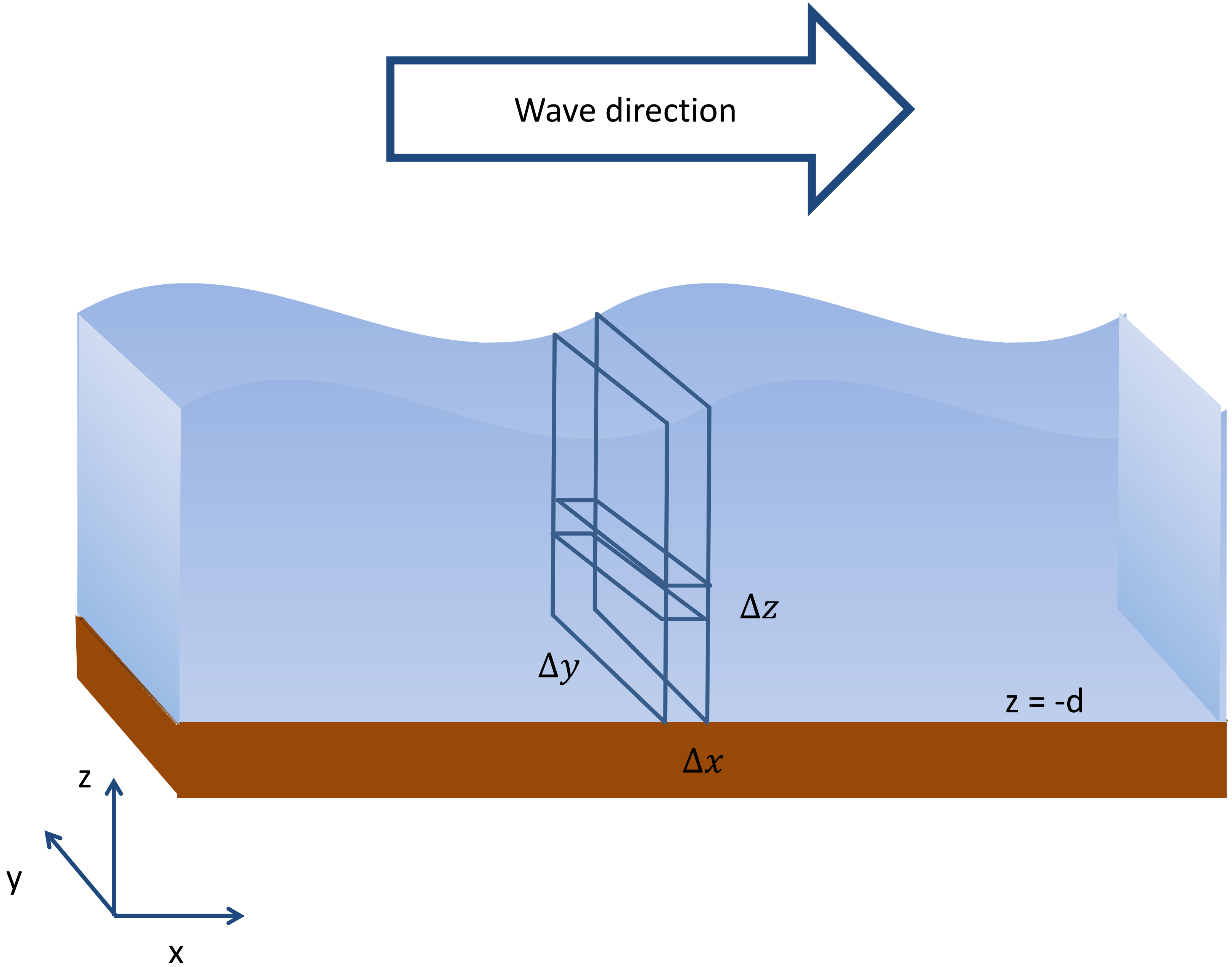
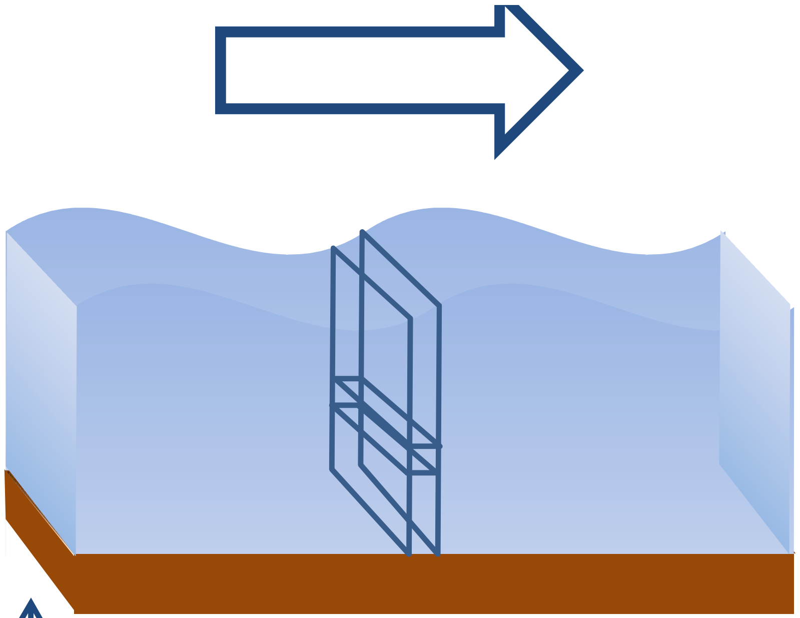
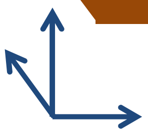
Radiation stress
Wave set up



Wave direction
Δz
Δy
z = -d
Δx
z
y
x



wave induced x-momentum
(integrating from bottom to sea surface)



$$q_x = \left(\int_{-d}^{\eta} \rho u_x dz\right) \Delta x \Delta y$$: wave induced x-momentum

$\downarrow$

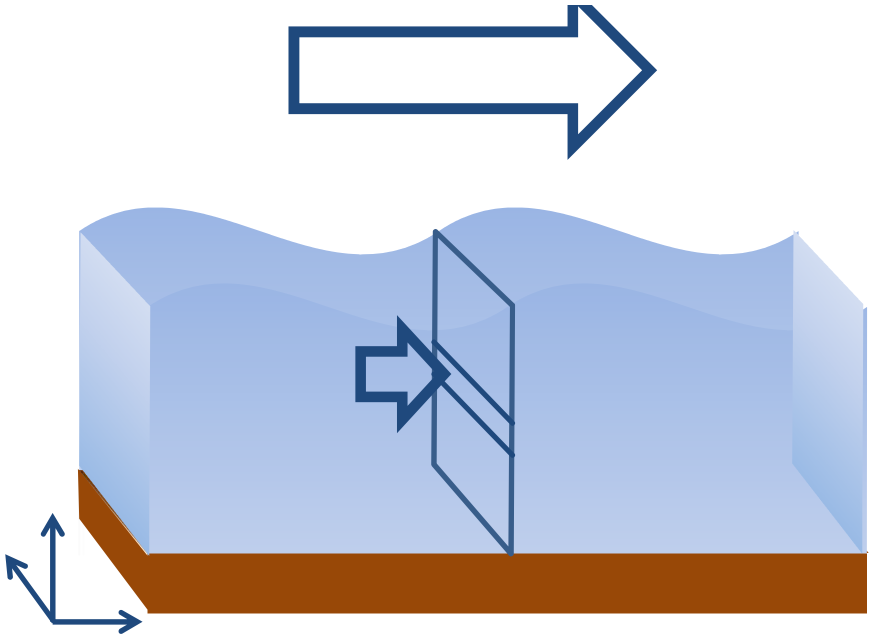
$$Q_x = \overline{\int_{-d}^{\eta} \rho u_x dz}$$:per unit surface area and averaged area

$$= \overline{\int_{-d}^{0} \rho u_x dz} + \overline{\int_{0}^{\eta} \rho u_x dz}$$: first term (averaged over time) is 0

$\downarrow$: linear wave theory

$$= \overline{\int_0^{\eta} \rho \left(wa \frac{\cosh[k(d+z)]}{\sinh(kd)} \sin(wt - kx) \right) dz}$$

$$= \frac{\rho a^2 w}{2 \tanh(kd)}$$



Transport of wave induced x-momentum
(integrating from bottom to sea surface)

↓

Radiation stress

(Rate of change of momentum is force : Newton 2nd law

therefor, rate of change of momentum per surface area is stress)

$s_{xx} = \left(\int_{-d}^{\eta} \rho u_x u_x + \ p_{wave} dz\right)\Delta y \Delta t$: total transport of vertical window

↓

first term : transport by particle motion $[N/m^2]$ second term : transport by wave induced pressure $[N/m^2]$

$$S_{xx} = \overline{\int_{-d}^{\eta} (\rho u_x u_x + \ p_{wave}) dz}$$: per y-unit and time, average over time

$$= \overline{\int_{-d}^{\eta} \rho {u_x}^2 dz} + \int_{-d}^{0} \overline{p_{wave}} dz + \overline{\int_{0}^{\eta} p_{wave} dz}$$: $\overline{p_{wave}} = -\overline{\rho {u_z}^2}$ (second order accuracy)

$$= \overline{\int_{-d}^{\eta} \rho {u_x}^2 dz} - \int_{-d}^{0} \overline{\rho {u_z}^2} dz + \overline{\int_{0}^{\eta} p_{wave} dz}$$: $\int_{-d}^{\eta} \rho {u_x}^2 dz = \int_{-d}^{0} \rho {u_x}^2 dz$ (second order accuracy)

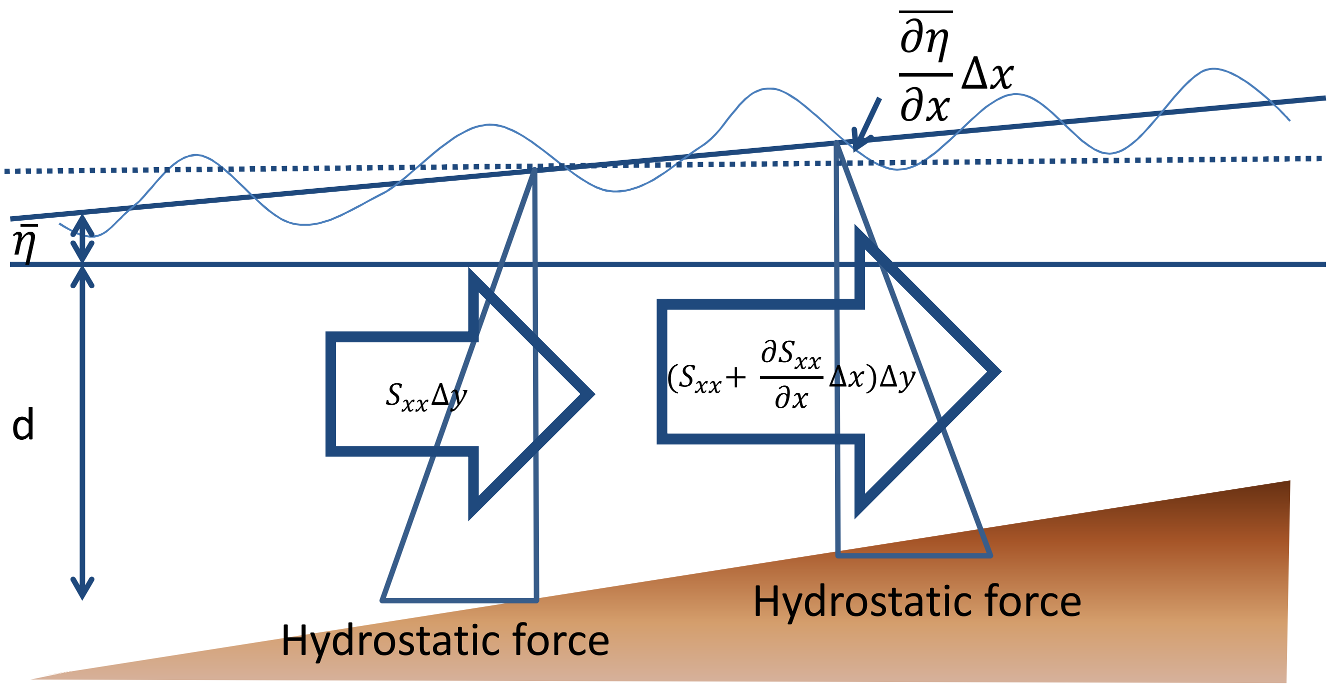
$$= \overline{\int_{-d}^{0} \rho {u_x}^2 dz} - \int_{-d}^{0} \overline{\rho {u_x}^2} dz + \overline{\int_{0}^{\eta} p_{wave} dz}$$

$$= \left(2n \ - \frac{1}{2}\right) E$$: $n = \frac{1}{2}\left(1 + \frac{2kd}{\sinh(2kd)}\right), \ E = \frac{1}{2}\rho g a^2$

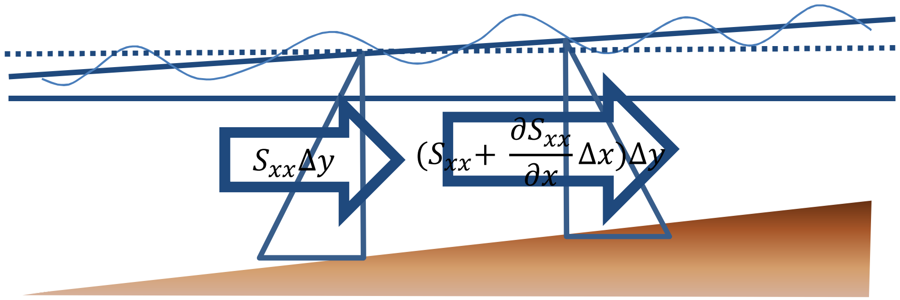
Wave-induced set up/down

Horizontal variations in the transport of wave-induced momentum (horizontal variations in radiation stress) may affect the mean sea level and generate current.

Simple case : a long-crested harmonic wave approaching beach in stationary situation, current is zero.



Wave-induced set up/down



Momentum balance equation

Storage of momentum Q_x = net import of Q_x + local production of Q_x

↓

$$\frac{\partial Q_x}{\partial t} = -\frac{\partial S_{xx}}{\partial x} - \rho g(d+\bar{\eta})\frac{\partial \bar{\eta}}{\partial x}$$

: $S_{xx} = Q_x u_x + p_{wave}$

: $-\rho g(d+\bar{\eta})\frac{\partial \bar{\eta}}{\partial x}$ hydrostatic force

$$0 = -\frac{dS_{xx}}{dx} - \rho g(d+\bar{\eta})\frac{\partial \bar{\eta}}{\partial x}$$

: stationary case

$$\frac{\partial \bar{\eta}}{\partial x} = \frac{-1}{\rho g d}\frac{dS_{xx}}{dx}$$

: $\bar{\eta} \ll d$

Slope of mean sea surface (set up/down) depends on change of radiation stress