入門線形代数

053M

問題 2-3

(2)
$$\left(\begin{array}{ccc|c} -3 & 3 & 1 & 1 \\ 1 & -1 & 2 & 0 \end{array}\right)$$
$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & -1 & 9 & 1 \\ 0 & 0 & 7 & 1 \end{array}\right) \begin{matrix} \\ (①+②\times 4) \\ (①+②\times 3) \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -\frac{2}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \begin{matrix} (①-②\times\frac{9}{7}) \\ (②/7) \end{matrix}$$ 基本

$$\begin{cases} x_1 - x_2 = -\frac{2}{7} \\ x_3 = \frac{1}{7} \end{cases} \quad \text{より} \quad \begin{pmatrix} x_1 = -\frac{2}{7} + C \\ x_2 = C \\ x_3 = \frac{1}{7} \end{pmatrix}$$

$$x = \begin{bmatrix} -\frac{2}{7} \\ 0 \\ \frac{1}{7} \end{bmatrix} + C\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad (C \in \mathbb{R})$$

(3)
$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ -1 & 0 & -3 & -4 \\ 1 & 2 & 7 & 3 \end{array}\right)$$
$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 0 & -1 & -2 & 1 \\ 0 & 3 & 6 & -2 \end{array}\right) \begin{matrix} \\ (①+②) \\ (③-①) \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 3 & 6 & -2 \end{array}\right) \begin{matrix} (①-②) \\ (-②) \\ \\ \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right) \quad (③-①\times 3)$$ 基本

$0=1$ となり 解なし //

(4)
$$\left(\begin{array}{ccc|c} 2 & -1 & 9 & 0 \\ -1 & 1 & -3 & 0 \\ 1 & -3 & -3 & 0 \end{array}\right) \quad (入れかえ)$$
$$\left(\begin{array}{ccc|c} 1 & -3 & -3 & 0 \\ 0 & -2 & -6 & 0 \\ 0 & 5 & 15 & 0 \end{array}\right) \begin{matrix} (③) \\ (②+③) \\ (①-③\times 2) \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & -3 & -3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{matrix} \\ (②\times(-\frac{1}{2})) \\ (掃) \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad (①+②\times 3)$$

基本

$$\begin{cases} x_1 + 6x_3 = 0 \\ x_2 + 3x_3 = 0 \end{cases} \quad \text{より} \quad \begin{pmatrix} x_1 = -6C \\ x_2 = -3C \\ x_3 = C \end{pmatrix}$$

$$x = C\begin{bmatrix} -6 \\ -3 \\ 1 \end{bmatrix} \quad (C \in \mathbb{R}) //$$

(5)
$$\left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 2 & 1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -5 & 4 & 1 & -6 \end{array}\right)$$
$$\left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & 1 & -1 & 1 & -5 & -7 \\ 0 & 3 & -3 & 3 & 3 & -3 \end{array}\right) \begin{matrix} \\ (②-①\times 2) \\ (①+③) \end{matrix}$$
$$\left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & 1 & -1 & 1 & -5 & -7 \\ 0 & 0 & 0 & 0 & 18 & 18 \end{array}\right) \quad (③-②\times 3)$$
$$\left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right) \begin{matrix} (①-\frac{1}{9}③) \\ (②-\frac{5}{18}③) \\ (③/18) \end{matrix}$$ 基本

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問題 2-3

1.(5)（続き）

$$\begin{cases} x_1 + 2x_3 - x_4 = 1 \\ x_2 - x_3 + x_4 = -2 \\ x_5 = 1 \end{cases}$$ から

$x_1 = 1 - 2c_1 + c_2$,　$x_2 = -2 + c_1 - c_2$

$x_3 = c_1$　$x_4 = c_2$　$x_5 = 1$ より

$$x = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + c_1 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$(c_1, c_2 \in \mathbb{R})$ //

(6)

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 4 & 5 & 1 \\ -1 & 2 & 0 & -1 & -2 & 0 \\ 3 & -6 & 1 & 4 & 7 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 3 & 3 & 3 & 1 \\ 0 & 0 & 8 & 8 & 8 & 2 \end{array}\right] \quad \begin{array}{l} \\ (①+②) \\ (①\times 3 - ③) \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 1 & 1 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{3} \end{array}\right] \quad \begin{array}{l} \\ (②/3) \\ (③ - ② \times \frac{8}{3}) \end{array}$$

☆ $0 = -\frac{2}{3}$ となるため

解なし //

(7)

$$\left[\begin{array}{ccccc|c} 1 & -4 & 3 & 4 & -3 & 0 \\ 1 & -2 & 0 & 1 & -2 & 0 \\ -1 & 2 & 2 & 1 & 4 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -4 & 3 & 4 & -3 & 0 \\ 0 & 2 & -3 & -3 & 1 & 0 \\ 0 & -2 & 5 & 5 & 1 & 0 \end{array}\right] \quad \begin{array}{l} \\ (② - ①) \\ (① + ③) \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -3 & -2 & -1 & 0 \\ 0 & 2 & -3 & -3 & 1 & 0 \\ 0 & 0 & 2 & 2 & 2 & 0 \end{array}\right] \quad \begin{array}{l} (① + ② \times 2) \\ \\ (② + ③) \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} (① + \frac{3}{2}③) \\ (② + \frac{3}{2}③) \\ (③/2) \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} \\ (②/2) \\ \end{array}$$ より

$$\begin{cases} x_1 + x_4 + 2x_5 = 0 \\ x_2 + 2x_5 = 0 \\ x_3 + x_4 + x_5 = 0 \end{cases}$$ で

$x_1 = -c_1 - 2c_2$,　$x_2 = -2c_2$

$x_3 = -c_1 - c_2$,　$x_4 = c_1$　$x_5 = c_2$ から

$$x = c_1 \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$(c_1, c_2 \in \mathbb{R})$ //