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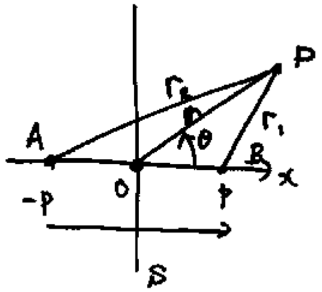
電磁気の考え方　3 (1)



$$\phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$r_1 = \sqrt{\left(\tfrac{1}{2}S\right)^2 + r^2 - 2\cdot\tfrac{1}{2}Sr\cos\theta}$$

$$= r\sqrt{\tfrac{1}{4}\left(\tfrac{S}{r}\right)^2 + 1 - \tfrac{S}{r}\cos\theta}$$

$$r_2 = \sqrt{\left(\tfrac{1}{2}S\right)^2 + r^2 + 2\cdot\tfrac{1}{2}Sr\cos\theta}$$

$$= r\sqrt{\tfrac{1}{4}\left(\tfrac{S}{r}\right)^2 + 1 + \tfrac{S}{r}\cos\theta}$$

$0 < x \ll 1$ で　$(1 \pm x)^{-\frac{1}{2}} \simeq 1 \mp \frac{1}{2}x$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{r}\left\{\left(1 + \tfrac{1}{2}\tfrac{S}{r}\cos\theta\right) - \left(1 - \tfrac{1}{2}\tfrac{S}{r}\cos\theta\right)\right\}$$

$$= \frac{S}{r^2}\cos\theta$$

$$\phi = \frac{qS}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{\mathbb{P}\cdot\mathbb{r}}{4\pi\varepsilon_0 r^3}$$

$$\phi_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{\mathbb{P}}{r^3}\cdot(\mathbb{r})_x + \mathbb{P}\cdot\mathbb{r}\left(\frac{1}{r^3}\right)_x\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{\mathbb{P}}{r^3}\cdot\begin{pmatrix}1\\0\end{pmatrix} + \mathbb{P}\cdot\mathbb{r}\,\frac{x}{r^5}\right](-3)\Big]$$

$$\mathbb{E} = -\mathrm{grad}\,\phi = \frac{1}{4\pi\varepsilon_0 r^3}\left[-\mathbb{P} + \frac{3(\mathbb{P}\cdot\mathbb{r})}{r^2}\mathbb{r}\right]$$

ここで　$\mathbb{P} = \begin{pmatrix}p\\0\end{pmatrix}$，$\mathbb{r} = \begin{pmatrix}x\\y\end{pmatrix}$，$x = r\cos\theta$，$y = r\sin\theta$

$$\mathbb{P}\cdot\mathbb{r} = px = pr\cos\theta$$

$$\mathbb{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left[-\begin{pmatrix}p\\0\end{pmatrix} + 3\frac{1}{r^2}pr\cos\theta\begin{pmatrix}x\\y\end{pmatrix}\right]$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\begin{pmatrix}-1 + 3\cos^2\theta\\ 3\sin\theta\cos\theta\end{pmatrix}$$

$\begin{pmatrix}E_r\\E_\theta\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}E_x\\E_y\end{pmatrix}$ だから

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$$\begin{pmatrix} E_r \\ E_\theta \end{pmatrix} = \frac{P}{4\pi\varepsilon_0 r^3} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -1 + 3\cos^2\theta \\ 3\sin\theta\cos\theta \end{pmatrix}$$

ここで $P = 3.0\times10^{-30}$, $r = 2.0\times10^{-8}$, $\theta = 30^\circ$ を代入すると

$$\begin{pmatrix} E_r \\ E_\theta \end{pmatrix} = \frac{3.0\times10^{-30}}{4\pi\frac{10^{-9}}{36\pi}(2.0\times10^{-8})^3} \frac{1}{2} \begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix} \begin{pmatrix} -1 + 3\left(\frac{\sqrt{3}}{2}\right)^2 \\ 3\cdot\frac{1}{2}\frac{\sqrt{3}}{2} \end{pmatrix}$$

$$= \frac{3^3}{2^3}\times10^{-30+9+24}\cdot\frac{1}{2}\cdot\frac{1}{4}\begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 5 \\ 3\sqrt{3} \end{pmatrix}$$

$$= \frac{3^3}{2^6}\times10^3\begin{pmatrix} 8\sqrt{3} \\ 4 \end{pmatrix}$$

$$= \frac{3^3}{2^4}\times10^3\begin{pmatrix} 2\sqrt{3} \\ 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 5.8457 \\ 1.6875 \end{pmatrix}\times10^3$$