問2．9

(1)

$$|S_1 \cup S_2 \cup S_3| = |S_1| + |S_2 \cup S_3| - |S_1 \cap (S_2 \cup S_3)|$$

$$= |S_1| + |S_2| + |S_3| - |S_2 \cap S_3| - |(S_1 \cap S_2) \cup (S_1 \cap S_3)|$$

$$= |S_1| + |S_2| + |S_3| - |S_2 \cap S_3| - (|S_1 \cap S_2| + |S_1 \cap S_3| - |S_1 \cap S_2 \cap S_1 \cap S_3|)$$

$$= |S_1| + |S_2| + |S_3| - (|S_1 \cap S_2| + |S_2 \cap S_3| + |S_3 \cap S_1|) + |S_1 \cap S_2 \cap S_3|.$$

(2)

$$|S_1 \cup S_2 \cup S_3| = |S_1| + |S_2| + |S_3| - (|S_1 \cap S_2| + |S_2 \cap S_3| + |S_3 \cap S_1|) + |S_1 \cap S_2 \cap S_3|$$

$$= \frac{n}{p_1} + \frac{n}{p_2} + \frac{n}{p_3} - \left(\frac{n}{p_1 p_2} + \frac{n}{p_2 p_3} + \frac{n}{p_3 p_1}\right) + \frac{n}{p_1 p_2 p_3}$$

$$= \frac{n}{p_1 p_2 p_3}(p_1 p_2 + p_2 p_3 + p_3 p_1 - (p_1 + p_2 + p_3) + 1).$$

(3)

$$\varphi(n) = n - |S_1 \cup S_2 \cup S_3|$$

$$= n\left(1 - \frac{1}{p_1 p_2 p_3}(p_1 p_2 + p_2 p_3 + p_3 p_1 - (p_1 + p_2 + p_3) + 1)\right)$$

$$= \frac{n}{p_1 p_2 p_3}(p_1 p_2 p_3 - (p_1 p_2 + p_2 p_3 + p_3 p_1) + (p_1 + p_2 + p_3) - 1)$$

$$= \frac{n}{p_1 p_2 p_3}(p_1 - 1)(p_2 - 1)(p_3 - 1)$$

$$= n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\left(1 - \frac{1}{p_3}\right).$$

問2．10

(1)

$2008 = 2^3 \times 251$であり，251は$2,3,5,7,11,13$で割り切れないので，素数．

よって，$\varphi(2008) = \varphi(2^3) \times \varphi(251) = 2^2(2-1) \times 250 = 1000$．

$\therefore 2008$と互いに素なものの個数は1000個．

(2)

$5460 = 2^2 \times 3 \times 5 \times 91$であり，$91$は$2,3,5,7$で割り切れないので，素数．

よって，$\varphi(5460) = \varphi(2^2) \times \varphi(3) \times \varphi(5) \times \varphi(91) = 2^1(2-1) \times 2 \times 4 \times 90 = 1440$．

$\therefore 5460$と互いに素なものの個数は1440個．