CHAPTER2 EXERCISES ANSWER

2.1.

(a)

i.(0,0,0,0,0)

ii.(1,0,0,0,0),(0,1,0,0,0),(0,0,1,0,0),(0,0,0,1,0),(0,0,0,0,1)

iii.(1,1,0,0,0),(1,0,1,0,0),(1,0,0,1,0),(1,0,0,0,1),(0,1,1,0,0),(0,1,0,1,0),(0,1,0,0,1),(0,0,1,1,0),(0,0,1,0,1),(0,0,0,1,1)

iv. (1,1,1,0,0),(1,1,0,1,0),(1,1,0,0,1),(1,0,1,1,0),(1,0,1,0,1),(1,0,0,1,1),(0,1,1,1,0),(0,1,1,0,1),(0,1,0,1,1),(0,0,1,1,1)

v. (1,1,1,1,0),(1,1,1,0,1),(1,1,0,1,1),(1,0,1,1,1),(0,1,1,1,1)

vi. (1,1,1,1,1)

あれ？位置の入れ替えもいるのか。面倒だから省略。

(b)

i.(0,0,0,0,0)

ii.(1,0,0,0,0),(0,1,0,0,0),(0,0,1,0,0),(0,0,0,1,0),(0,0,0,0,1)

iii.(1,1,0,0,0),(1,0,1,0,0),(1,0,0,1,0),(1,0,0,0,1),(0,1,1,0,0),(0,1,0,1,0),(0,1,0,0,1),(0,0,1,1,0),(0,0,1,0,1),(0,0,0,1,1)

iv. (1,1,1,0,0),(1,1,0,1,0),(1,1,0,0,1),(1,0,1,1,0),(1,0,1,0,1),(1,0,0,1,1),(0,1,1,1,0),(0,1,1,0,1),(0,1,0,1,1),(0,0,1,1,1)

v. (1,1,1,1,0),(1,1,1,0,1),(1,1,0,1,1),(1,0,1,1,1),(0,1,1,1,1)

vi. (1,1,1,1,1)

こっちは、(2.3) 式と一致。

2.2.

(a)20

(b)6

(c)120

(d)270725

2.3.

(a)p=1/2

x=0 1/64

x=1 6/64 = 3/32

x=2 15/64

x=3 20/64 = 5/16

x=4 15/64

x=5 6/64 = 3/32

x=6 1/64

mean = np = 3

standard deviation = $\sqrt{np(1-p)} = \sqrt{3/2} = 1.22$

(b)p=1/6

x=0 $(5/6)^6 = 15625/46656$

x=1 $6(1/6)(5/6)^5 = 18750/46656 = 3125/7776$

x=2 $15(1/6)^2(5/6)^4 = 9375/46656 = 3125/15552$

x=3 $20(1/6)^3(5/6)^3 = 2500/46656 = 625/11664$

x=4 $15(1/6)^4(5/6)^2 = 375/46656 = 125/15552$

x=5 $6(1/6)^5(5/6) = 30/46656 = 5/7776$

x=6 $(1/6)^6 = 1/46656$

mean = np = 1

standard deviation = $\sqrt{np(1-p)} = \sqrt{5/6} = 2.45$

2.4.

なんだこれ。意味わからん。確率分布が規格化されてないぞ。

2.5.

$y = x - 1$,$m = n - 1$

$$\begin{aligned}
\mu &= \sum_{x=0}^{n}\left[x\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] & (1)\\
&= \sum_{x=1}^{n}\left[x\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] & (2)\\
&= \sum_{x=1}^{n}\left[\frac{n!}{(x-1)!(n-x)!}p^x(1-p)^{n-x}\right] & (3)\\
&= \sum_{y=0}^{m}n\left[\frac{m!}{y!(m-y)!}p^{y+1}(1-p)^{m-y}\right] & (4)\\
&= np\sum_{y=0}^{m}\left[\frac{m!}{y!(m-y)!}p^{y}(1-p)^{m-y}\right] & (5)\\
&= np & (6)
\end{aligned}$$

途中で、Hint の式を使った。
2.6.
確率は n=3,p=1/10 の二項分布
lemon=0

$$\frac{3!}{0!3!}(1/10)^0(9/10)^3 = 729/1000 \quad (7)$$

lemon=1

$$\frac{3!}{1!2!}(1/10)^1(9/10)^2 = 243/1000 \quad (8)$$
$$1 \div 243/1000 = 4.1 \quad (9)$$

lemon=2

$$\frac{3!}{2!1!}(1/10)^2(9/10)^1 = 27/1000 \quad (10)$$
$$1 \div 27/1000 = 37 \quad (11)$$

lemon=3

$$\frac{3!}{3!0!}(1/10)^3(9/10)^0 = 1/1000 \quad (12)$$
$$1 \div 1/1000 = 1000 \quad (13)$$

2.7.
$y = x - 1$,$m = n - 1$

$$\sigma^2 = \sum_{x=0}^{n}\left[(x-\mu)^2\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] \tag{14}$$

$$= \sum_{x=0}^{n}\left[x^2\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] - 2\mu\sum_{x=0}^{n}\left[x\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] + \mu^2\sum_{x=0}^{n}\left[\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] \tag{15}$$

$$= \sum_{x=1}^{n}\left[x^2\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\right] - 2\mu\times\mu + \mu^2 \tag{16}$$

$$= \sum_{y=0}^{m}np\left[(y+1)\frac{m!}{y!(m-y)!}p^y(1-p)^{m-y}\right] - \mu^2 \tag{17}$$

$$= \sum_{y=0}^{m}np\left[y\frac{m!}{y!(m-y)!}p^y(1-p)^{m-y}\right] + \sum_{y=0}^{m}np\left[\frac{m!}{y!(m-y)!}p^y(1-p)^{m-y}\right] - \mu^2 \tag{18}$$

$$= np\times mp + np - n^2p^2 \tag{19}$$

$$= n(n-1)p^2 + np - n^2p^2 \tag{20}$$

$$= np - np^2 \tag{21}$$

$$= np(1-p) \tag{22}$$

2.8.
n=1035,p=1/4 の二項分布と考えると、$\mu = np = 258.75$ となる。
283-258.75 = 24.25,258.75-24.25=234.5
よって、誤差が小さくなるのは、235 から 282 まで。

$$1 - \sum_{x=235}^{282}\frac{1035!}{x!(1035-x)!}(1/4)^x(3/4)^{1035-x} = 0.0848 \tag{23}$$

=2.8 program=

```
import math

def factorial(n, i):
x = 1
while n > i:
x = x*n
n = n-1
return x

x = 0
for i in range(235,283):
x = x + factorial(1035,1035-i)/factorial(i,0)*pow(0.25,i)*pow(0.75,1035-i)
print(1-x)
```

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2.9.
(a)$\mu = np = 32 \times 0.073 = 2.3$
(b)

$$1 - \sum_{x=0}^{4}\frac{32!}{x!(32-x)!}(0.073)^x(0.927)^{32-x} = 0.0803 \tag{24}$$

==2.9 program==

```
import math
```

```
def factorial(n, i):
x = 1
while n > i:
x = x*n
n = n-1
return x

x = 0
for i in range(0,5):
x = x + factorial(32,32-i)/factorial(i,0)*pow(0.073,i)*pow(0.927,32-i)
print(1-x)
```

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2.10.

めんどくさいので省略

2.11.

$$
\begin{aligned}
P_P(\mu;\mu) &= \frac{\mu^\mu}{\mu!}e^{-\mu} &(25)\\
&= \frac{\mu^{\mu-1}}{(\mu-1)!}e^{-\mu} &(26)\\
&= P_P(\mu-1;\mu) &(27)
\end{aligned}
$$

2.12.

$$
\begin{aligned}
\sigma^2 &= \sum_{x=0}^{\infty}\left[(x-\mu)^2\frac{\mu^x}{x!}e^{-\mu}\right] &(28)\\
&= \sum_{x=0}^{\infty}\left[x^2\frac{\mu^x}{x!}e^{-\mu}\right] - 2\mu\sum_{x=0}^{\infty}\left[x\frac{\mu^x}{x!}e^{-\mu}\right] + \mu^2\sum_{x=0}^{\infty}\left[\frac{\mu^x}{x!}e^{-\mu}\right] &(29)\\
&= \sum_{y=0}^{\infty}\left[(y+1)\frac{\mu^{y+1}}{y!}e^{-\mu}\right] - 2\mu\times\mu+\mu^2 &(30)\\
&= \mu\sum_{y=0}^{\infty}\left[y\frac{\mu^y}{y!}e^{-\mu}\right] + \mu\sum_{y=0}^{\infty}\left[\frac{\mu^y}{y!}e^{-\mu}\right] - \mu^2 &(31)\\
&= \mu\times\mu+\mu-\mu^2 &(32)\\
&= \mu &(33)
\end{aligned}
$$

2.13.

(a) $S_P = 1 - e^{-2}\sum_{x=0}^{7}\frac{2^x}{x!} = 0.00110$

===2.13a program===

```
import math

def factorial(n, i):
x = 1
while n > i:
x = x*n
n = n-1
return x

x = 0
```

```
for i in range(0,8):
x = x + pow(math.e,-2)*pow(2,i)/factorial(i,0)
print(1-x)
```

==================

(b) 10 分で平均は、2/24/6 = 0.0139 だから、

$S_P = 1 - e^{-0.0139}\sum_{x=0}^{7}\frac{0.0139^x}{x!} = 1.11e-16$

```
==2.13b program ==
import math

def factorial(n, i):
x = 1
while n > i:
x = x*n
n = n-1
return x

x = 0
for i in range(0,8):
x = x + pow(math.e,-0.0139)*pow(0.0139,i)/factorial(i,0)
print(1-x)
```

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2.14.

(a) 二項分布で計算？$p = 670/1000 = 0.670$ より、

$\sigma = \sqrt{1000(0.670)(0.33)} = 14.9$ から、

$$N_R = 670 \pm 14.9 \tag{34}$$

$$N_L = 330 \pm 14.9 \tag{35}$$

(b)$((670 \pm 14.9) - (330 \pm 14.9))/1000 = (340 \pm 29.8)/1000 = 0.340 \pm 0.0298$

(c)A= 0.400 とすると、$(1000 - 2N_L)/1000 = 0.400$,$N_L = 300, N_R = 700$

$p = 0.700$ より、

$\sigma = \sqrt{1000(0.700)(0.300)} = 14.5$ から、

$$N_R = 670 \pm 14.5 \tag{36}$$

$$N_L = 330 \pm 14.5 \tag{37}$$

$$A = ((670 \pm 14.5) - (330 \pm 14.5))/1000 = (340 \pm 29.0)/1000 = 0.340 \pm 0.029 \tag{38}$$

2.15.

(a) ポアソン分布より、

$$\bar{x} = (1 \times 10^6)/(200 \times 10^{-9}) \tag{39}$$

$$= 0.2 = \mu \tag{40}$$

$$\sum_{x=1}^{\infty} P_P = 1 - \frac{\mu^0}{0!}e^{-\mu} \tag{41}$$

$$= 1 - e^{-\mu} \tag{42}$$

$$\text{efficiency} = (1 - e^{-\mu})/\mu \tag{43}$$

(b) てか、これよくわからん

2.16.

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{1/2\Gamma}{\sigma}\right)^2\right] = \frac{1}{2}\frac{1}{\sigma\sqrt{2\pi}} \tag{44}$$

$$\exp\left[-\frac{1}{2}\left(\frac{1/2\Gamma}{\sigma}\right)^2\right] = 1/2 \tag{45}$$

$$\tag{46}$$

$\Gamma = \alpha\sigma$ とすると、

$$\exp\left[-1/8\alpha^2\right] = 1/2 \tag{47}$$

$$\alpha = \sqrt{-8\log(1/2)} = \sqrt{8\log 2} = 2.354 \tag{48}$$

2.17.

$$\begin{aligned}
\bar{r} &= \int_0^\infty rCr^2e^{-r/R}dr && (49)\\
&= \int_0^\infty Cr^3e^{-r/R}dr && (50)\\
&= [-Cr^3Re^{-r/R}]_0^\infty + \int_0^\infty 3Cr^2Re^{-r/R}dr && (51)\\
&= [-3Cr^2R^2e^{-r/R}]_0^\infty + \int_0^\infty 6CrR^2e^{-r/R}dr && (52)\\
&= [-6CrR^3e^{-r/R}]_0^\infty + \int_0^\infty 6CR^3e^{-r/R}dr && (53)\\
&= [-6CR^4e^{-r/R}]_0^\infty = 6CR^4 && (54)
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= \int_0^\infty (r-\bar{r})^2Cr^2e^{-r/R}dr && (55)\\
&= \int_0^\infty (r-6CR^4)^2Cr^2e^{-r/R}dr && (56)\\
&= \int_0^\infty Cr^4e^{-r/R}dr - 12CR^4\int_0^\infty Cr^3e^{r/R}dr + 36C^2R^8\int_0^\infty Cr^2e^{-r/R}dr && (57)\\
&= 24CR^5 - 72C^2R^8 + 72C^3R^{11} && (58)\\
\sigma &= \sqrt{24CR^5 - 72C^2R^8 + 72C^3R^{11}} && (59)
\end{aligned}$$

規格化を考える。

$$\int_0^\infty Cr^2e^{-r/R}dr = 2CR^3 = 1 \tag{60}$$

$$C = \frac{1}{2R^3} \tag{61}$$

2.18.

$$\begin{aligned}
f(z) &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right) && (62)\\
\frac{df(z)}{dz} &= \frac{1}{\sqrt{2\pi}}(-z)\exp\left(-\frac{z^2}{2}\right) && (63)\\
\frac{d^2f(z)}{dz^2} &= \frac{1}{\sqrt{2\pi}}\left[-\exp\left(-\frac{z^2}{2}\right) + z^2\exp\left(-\frac{z^2}{2}\right)\right] && (64)\\
&= \frac{1}{\sqrt{2\pi}}(z^2-1)\exp\left(-\frac{z^2}{2}\right) = 0 && (65)
\end{aligned}$$

$z=\pm1$ で極値を取る。すなわち、$x=\mu\pm\sigma$ のときである。このとき、接線は、(対称性より z=1 の場合を考える)

$$y-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\right)=-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\right)(z-1) \tag{66}$$

すると、$y=0$ のとき、

$$0=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\right)(z-2) \tag{67}$$

よって、$z=2$ つまり、$x=\mu+2\sigma$ で x 軸と交わる。