これが~~状態方程式~~である．伝達関数を $H(s)$ とすると，

入出力方程式

$$H(s) = \frac{3s+84}{s^2+s-6} = -\frac{15}{s+3} + \frac{18}{s-2} \tag{21}$$

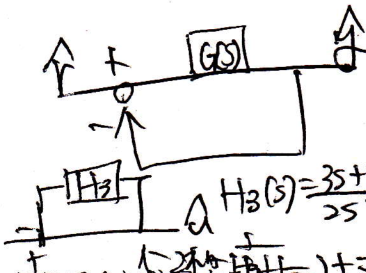
$$H(s)\frac{1}{s} = \frac{3s+84}{(s^2+s-6)s} = -\frac{14}{s} + \frac{5}{s+3} + \frac{9}{s-2} \tag{22}$$

となるので，インパルス応答 $y_I(t)$ とステップ応答 $y_S(t)$ はそれぞれ次になる．

$$y_I(t) = -15e^{-3t} + 18e^{2t}, \tag{23}$$

$$y_S(t) = -14 + 5e^{-3t} + 9e^{2t}. \tag{24}$$

問 5



$\exists s \; |G(s)| < 1$

$\hat{y} = G(s)(\hat{r} - \hat{y})$

$H_3(s) = \frac{3s+1}{2s+1}$

$$\hat{y} = H_2(s)\hat{u} \tag{25}$$

$(1 + H_1(s)H_3(s))\hat{u} = H_1(s)(\hat{r} - \hat{y})$

$\hat{u} = \frac{H_1(s)(\hat{r}-\hat{y})}{1 + H_1(s)H_3(s)}$

$\hat{y} = \frac{1}{1+G(s)}\hat{r}$

である．ここで $\hat{u}$ に注目すると，

$$\hat{u} = H_1(s)\{(\hat{r}-\hat{y}) - H_3(s)\hat{u}\} \quad \Rightarrow \quad \hat{u} = \frac{H_1(s)}{1+H_1(s)H_3(s)}(\hat{r}-\hat{y}) \tag{26}$$

となるので，

$$\hat{y} = \frac{H_1(s)H_2(s)}{1+H_1(s)H_3(s)}(\hat{r}-\hat{y}) \tag{27}$$

$$\Rightarrow \quad \hat{y} = \frac{H_1(s)H_2(s)}{1+H_1(s)H_2(s)+H_1(s)H_3(s)}\hat{r} \tag{28}$$

$\hat{y} = \frac{H_1(s)H_2(s)}{1+H_1(s)H_3(s)}(\hat{r}-\hat{y})$

を得る．分子と分母はそれぞれ

$H_1(s)H_2(s) = \frac{4}{(s+1)(4s+1)}$

$H_1(s)H_3(s) = \frac{4(3s+1)}{(4s+1)(2s+1)}$

$$H_1(s)H_2(s) = \frac{4}{(s+1)(4s+1)}, \tag{29}$$

$$1 + H_1(s)H_2(s) + H_1(s)H_3(s) = \frac{4s^3+21s^2+22s+5}{(s+1)(4s+1)(2s+1)} \tag{30}$$

となるので，次の関係式を得る．

$1 + \frac{4}{(s+1)(4s+1)} + \frac{4(3s+1)}{(4s+1)(2s+1)}$

$$\hat{y} = \frac{4(2s+1)}{4s^3+21s^2+22s+5}\hat{r} \tag{31}$$

$= \frac{(4s+1)(2s+1)(s+1) + 4(2s+1) + 4(3s+1)(s+1)}{(4s+1)(2s+1)(s+1)}$

$= \frac{(4s+1)(2s^2+3s+1) + 8s+4 + 4(3s^2+4s+1)}{(4s+1)(2s+1)(s+1)}$

$= \frac{8s^3 + 12s^2 + 4s + 2s^2 + 3s + 1 + 8s + 4 + 12s^2 + 16s + 4}{(4s+1)(2s+1)(s+1)}$